Modern Physics—PHYS 220
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Lab: Michelson Interferometry

Josh Diamond & John Cummings

Theory

This experiment explores the use of the Michelson interferometer to determine the wavelength of visible light and perform other precision measurements.

The basic principle of all interferometry measurements is that if two coherent light beams originate from a common source and travel different paths to a common end point, interference will occur at the end point. The nature of the interference is determined by the difference in the optical path lengths. A schematic diagram of a Michelson interferometer is shown in Figure 1 below.

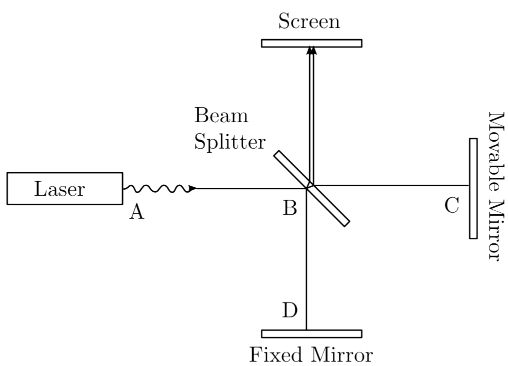

Figure 1: Michelson Interferometer

A monochromatic light beam enters the interferometer along line AB. At B the beam encounters a half-silvered mirror which functions as a beam splitter. Part of the beam continues along the same direction, reflects from the movable mirror at C and returns to B. The remainder of the beam reflects from the beam splitter, strikes the fixed mirror at D and returns to the beam splitter at B. The two beams recombine at B and travel together to the screen.

During the time when the beams are split, they travel different paths. One path is of length equal to twice the distance from B to C, denoted $\langle BC \rangle$. The other path length is twice the distance from B to D, denoted $\langle BD \rangle$. The difference between these path lengths determines the nature of the interference seen at a particular point on the screen. If we let the difference in path lengths be ΔL, we have

$$\Delta L = 2\langle BC\rangle - 2\langle BD\rangle \tag{1}$$

then the condition for constructive interference is

$$\Delta L = m\lambda \tag{2}$$

and the condition for destructive interference is

$$\Delta L = (m + 1/2)\lambda. \tag{3}$$

In general the interference pattern on the screen will consist of alternating bright and dark regions (usually fine curved lines) called "fringes".[1]

[1] In Eqs. 2 & 3, λ is the light wavelength and m is an integer (0, 1, 2, ...).

Turning the micrometer dial on the interferometer displaces the moveable mirror, changing the distance $\langle BC\rangle$, while $\langle BD\rangle$ remains fixed. Thus, the path length difference ΔL between the two beams can be varied in a controlled manner. If the distance $\langle BC\rangle$ changes from an initial value of $\langle BC\rangle_1$ to a final value $\langle BC\rangle_2$, then, using Eq. 1, the change in the path length difference ΔL is given by:

$$\Delta L_2 - \Delta L_1 = 2\langle BC\rangle_2 - 2\langle BD\rangle - (2\langle BC\rangle_1 - 2\langle BD\rangle). \tag{4}$$

Hence,

$$\Delta L_2 - \Delta L_1 = 2(\langle BC\rangle_2 - \langle BC\rangle_1) = 2(d_2 - d_1), \tag{5}$$

where d_1 is the initial micrometer reading and d_2 is the final micrometer reading.

Each time ΔL changes by one wavelength λ, the integer m changes by one unit (see Eqs. 2 & 3), and the observed pattern of interference fringes shifts by one fringe. As the micrometer reading changes from d_1 to d_2, m will change from m_1 to m_2. Combining Eqs. 2 and 4, we can express

$$\begin{aligned}\Delta m &= m_2 - m_1 \\ &= \frac{\Delta L_2}{\lambda} - \frac{\Delta L_1}{\lambda} \\ &= \frac{2(d_2 - d_1)}{\lambda}\end{aligned}$$

so

$$\boxed{\Delta m = \frac{2(d_2 - d_1)}{\lambda}} \tag{6}$$

where Δm is the number of fringes shifted past a fixed point as the micrometer dial reading changes from d_1 to d_2.

On this apparatus, the micrometer dial is designed so that one complete revolution of the micrometer dial corresponds to a change $d_2 - d_1 = 25\ \mu\text{m}$.[2]

[2] Note: $1\ \mu\text{m} = 10^{-6}$ m.

Index of refraction of a glass slide

It is difficult to measure the index of refraction of a thin slab of transparent material by observing refraction, because a beam refracted within the material will return to its original direction after exiting the slab.[3] Although the beam is shifted slightly parallel to itself, this effect is not normally easily measurable.

[3] Be sure you understand this statement by making a sketch. Check your work with your instructor before proceeding.

An attractive solution to this problem is to measure the fringe shift occurring when the slab is rotated in the path of one of the interferometer beams–while the other beam is unchanged. This rotation causes the light beam to traverse a thicker portion of the material, as the slab is turned away from being perpendicular to the light beam. Since the light wavelength is shorter in the slab than in air[4], the beam passing through the slab will contain increasingly more wavelengths as the angle from the perpendicular orientation increases. For each additional wavelength contained in the slab (compared to an equal thickness of air), the interference fringe pattern will shift by one fringe.

[4] Why?

Thus, there is a relationship between the following quantities:

n the index of refraction of the slab

t the thickness of the slab

θ the angle through which the slab is rotated, relative to an initial position in which the slab is perpendicular to the incident light beam

Δm the number of fringes passing a fixed point when the slab is rotated by angle θ from its initial position.

To a good approximation, it can be shown that this relationship is:

$$\boxed{n = \frac{t - \lambda(\Delta m/2)}{t - [\lambda(\Delta m/2)/(1 - \cos\theta)]}} \quad (7)$$

Index of refraction of a gas

If one of the interferometer beams travels through a sealed chamber, that beam will travel more rapidly inside the chamber than it would outside the chamber if the pressure of the air inside the chamber is less than normal atmospheric pressure (i.e. closer to being a vacuum).

This means that the index of refraction n of air actually depends on the air pressure P, and we shall denote this index of refraction as n_P . Since the light frequency is unaffected by changes in the medium, the wavelength of the light being used depends on the index of refraction in the same way as the velocity of light, i.e.

$$\lambda_P = \frac{\lambda}{n_P} \quad (8)$$

where λ is the wavelength of the light in a vacuum and λ_P is the wavelength in air at pressure P.

In this experiment, we form the usual interference pattern with the two interferometer beams, but now with one of the beams traveling through the air chamber. At the initial atmospheric pressure, the number of wavelengths of light contained in the air chamber is equal to $2t/\lambda_{\text{atm}}$, where t is the thickness of the air layer in the chamber measured along the light path.[5] As the pressure inside the chamber decreases, n_P decreases, λ_P increases, and fewer wavelengths are contained inside the chamber. The change in the number of wavelengths in the chamber as the pressure is varied will equal the number Δm of fringes that move past a fixed point in the interference pattern.

[5] Where does the factor of two come from?

In the experiment, we begin with the air pressure in the chamber equal to atmospheric pressure, so that the index of refraction initially is n_{atm}. The pressure is then decreased to a value P. The number of wavelengths inside the chamber decreases to $2t/\lambda_P$, and it follows that

$$\begin{aligned}\Delta m &= \frac{2t}{\lambda_{\text{atm}}} - \frac{2t}{\lambda_P} \\ &= \frac{2t}{\lambda}(n_{\text{atm}} - n_P),\end{aligned} \tag{9}$$

where Δm is the number of fringes passing a fixed point while the gas pressure in the chamber varies from atmospheric pressure to a pressure P, and where the second equality was obtained using Eq. 8. [6]

[6] Verify for yourself the mathematical derivation between the first and last step in Eq. 9. Check with your instructor before proceeding.

At a final gas pressure of zero in the chamber (a perfect vacuum), $n_P = 1$, and Eq. 9 becomes

$$\boxed{(\Delta m)_{P=0} = \frac{2t}{\lambda}(n_{\text{atm}} - 1)} \tag{10}$$

Although $(\Delta m)_{P=0}$ is not directly measurable (because we cannot obtain a perfect vacuum in the chamber), it can be determined from the other Δm measurements by graphical extrapolation. Knowing it, we can then use Eq. 10 to find n_{atm}, the index of refraction of air at atmospheric pressure.

Preliminary Question

1. A monochromatic light source is used in a Michelson interferometer. The micrometer dial controlling the moveable mirror is rotated until 100 interference fringes pass a fixed point. The difference between the final and initial micrometer readings is 32.0 microns.[7] Use the given information to determine the wavelength of the light.

[7] The 32.0 microns is the displacement of the moveable mirror.

Equipment

PASCO Michelson interferometer, laser, lens, screen, mounted glass slide, micrometer, and gas cell.

Procedure

Caution: IN THOSE PORTIONS OF THE EXPERIMENT USING A LASER LIGHT SOURCE, BE CAREFUL NOT TO LOOK DIRECTLY INTO THE BEAM OR TO LET A DIRECT REFLECTION OF THE BEAM ENTER YOUR EYE.

1. Adjustment of Michelson interferometer

   (a) Use a He-Ne laser beam with the interferometer set up as in Fig. 1. By adjusting the orientation of the fixed mirror, align the two recombined interferometer beams so that they are superimposed on the screen.

   (b) Insert a lens between the laser and the beam splitter to broaden the laser beam. Observe the interference fringes projected on a screen.

   (c) Carefully adjust the lens and the orientation of the fixed mirror to obtain an interference pattern of concentric rings (resembling a target with a "bulls-eye").[8]

[8] Explain why a ring pattern is formed. *Hint: What is the shape of the laser light wave fronts when the lens is present?*

2. Measuring the wavelength of a He-Ne laser

   The wavelength of the laser can be determined by using the micrometer dial to displace the movable mirror by a known amount and counting the corresponding number of fringes that pass by on the screen. Count a fairly large number of fringes, say roughly 30, and note the micrometer reading before and after. Repeat this measurement several times using different parts of the micrometer dial to reduce errors from nonlinearities in the mirror mechanism.

3. Qualitative observations

   (a) What happens to the interference pattern when an opaque sheet is placed in the path of one of the two beams produced by the interferometer before they recombine. Why?

   (b) What happens to the interference pattern when the table holding the interferometer is tapped or lightly jarred? Explain the sensitivity of the pattern to these perturbations.

4. Index of refraction of a glass slide[9]

   (a) We use the laser light source and a glass slide as a sample. Mount the glass slide on a magnetic holder placed at the center of a table, which allows it to be rotated about a vertical axis. The angle of rotation can be read using the scale printed on the interferometer base. The interferometer is designed so that slide is in the path of the laser beam passing between the beam splitter and the moveable mirror of the interferometer (path BC in Fig. 1).

[9] Note: the micrometer dial controlling the moveable mirror is not used in this part of the experiment.

(b) Align the slide initially so that its plane is perpendicular to the laser beam passing through it. The fringe pattern itself can be used to determine this position—it is the point at which the fringes begin to reverse direction as the slide is slowly rotated.[10]

[10] Why do the fringes reverse direction at this point? *Suggestion: To accomplish the alignment, set the rotating table so that its arm is accurately at zero on the angular scale. Then, without allowing the table to move, carefully rotate the mount holding the glass slide until you observe the reversal of the fringe motion. Position the slide at the turning point just before the reversal begins.*

(c) Slowly rotate the table by moving its arm (without touching the slide), counting the number of fringes passing a fixed point. Continue until you reach 100 fringes, or until you reach the maximum calibrated angle (20 degrees)—whichever comes first. Record the number of fringes counted and the angle θ through which the slide has been rotated.[11] Also measure the thickness of the slide with a micrometer to three significant figures.

[11] θ should be determined to the nearest tenth of a degree.

5. Index of refraction of a gas[12]

[12] Note: the micrometer dial controlling the moveable mirror is not used in this experiment.

Mount the gas cell so that one of the interferometer beams passes through it, perpendicular to its side windows. Count the fringes passing a fixed point as the pressure in the chamber is slowly reduced from atmospheric to the minimum value obtainable. Record the *cumulative number of fringes passing* (starting from atmospheric pressure) for each increment of 10 cmHg of pressure decrease.[13] Also measure the thickness t of the air space in the chamber along the light path.

[13] Note that the pressure gauge is actually a vacuum gauge, and gives a reading of the *pressure decrease* $\Delta P = P_{\text{atm}} - P$ in cmHg, where P_{atm} is atmospheric pressure and P is the pressure in the chamber. P_{atm} is approximately equal to 76 cmHg, and can be determined more precisely from a barometer.

Analysis

1. Compute the wavelength of the laser light using the data you collected in procedure step 2 using Eq. 6.[14]

[14] Note: For a He-Ne Laser light wavelength in a vacuum $\lambda = 632.8$ nm.

2. Compute, using Eq. 7, the index of refraction of the glass slide studied in procedure step 4. Compare to typical tabulated values of the index of refraction of glass.[15]

[15] Note: Eq. 7 is complicated and care must be taken in the computation. Do not round off numerical data values or any intermediate numerical results. How many significant figures should your final result for n have?

3. Plot the cumulative number of fringes Δm passing as a function of the pressure decrease $\Delta P = P_{\text{atm}} - P$ for your data taken with the gas cell. Use Python (or your favorite programming language) to fit a linear model through your data. Use the intercept of your fit to infer $(\Delta m)_{P=0}$, the number of fringes at $P = 0$ (i.e., to pressure decrease $\Delta P = P_{\text{atm}}$).

   Then use Eq. 10 to calculate the index of refraction of air at atmospheric pressure, n_{atm}. Compare this result to a tabulated value.[16]

[16] Suggestion: First compute $n_{atm} - 1$ using Eq. 10. Then you can easily write the result for n_{atm} itself. Note: The calculated n_{atm} will be slightly greater than 1, and *the final result must not be rounded off to 1.0!*

Uncertainty Analysis

1. Measuring the wavelength: There are two measurements: the change in the number of fringes Δm, and the displacement of the movable mirror, $\Delta d = d_2 - d_1$, as measured by the micrometer. Make at least 2-3 different measurements, or two different ranges of displacement. The expression for wavelength λ is a simple quotient, so the uncertainty in the wavelength is a simple addition of the uncertainties in quadrature:

$$\sigma_\lambda = \lambda\sqrt{\left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_{\Delta d}}{\Delta d}\right)^2}.$$

Note that if one of these fractions is significantly smaller than the other, you can omit it.[17] Be sure to check that the experimental value for λ you obtain agrees with your expectation for red light, and with your answer to the preliminary question.

[17] Why?

2. Measuring the index of refraction of a glass slide: The expression for n_{glass} is an approximation and is somewhat complex. To simplify the uncertainty analysis, estimate uncertainty in the slide thickness, t, the change in the number of fringes, Δm, and the wavelength, λ, and find the relative uncertainty in each.[18] Now, the angle θ appears in the equation not directly as the angle, but as $\cos\theta$; however, note that

$$\sigma(\cos\theta) = \left|\frac{\partial\cos\theta}{\partial\theta}\right| d\theta \approx \sin\theta,$$

and so the uncertainty in n_{glass} becomes

$$\sigma_n = n\sqrt{\left(\frac{\sigma_t}{t}\right)^2 + \left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_\lambda}{\lambda}\right)^2 + (\tan\theta)^2}.$$

Of course, if one or more of these fractions is significantly smaller than the other, you can omit it.

[18] In other words, divide the uncertainty by the measurement.

3. Measuring index of air: Estimate the uncertainty in the measured uncertainties in the thickness t, the wavelength λ, and the change in the number of fringes, Δm. Use a similar addition-in-quadrature developed above in order to estimate the uncertainty in $(n_{atm} - 1)$.