

Modern Physics—PHYS 220

Fall 2025

Lab: Electron Diffraction

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Theory

The purpose of this experiment is to demonstrate the wave nature of electrons. A beam of electrons accelerated by a potential of several kilovolts is Bragg-diffracted by a polycrystalline graphite film, and the resulting ring pattern is analyzed to study the relationship of the electron wavelength to its energy.

Electron Wavelength

A fundamental principle of quantum mechanics is the wave-particle duality, which states that on the atomic level an object such as electron, originally thought of as particle, can also exhibit wave behavior. Also, light, typically thought of as a wave, can exhibit particle (photon) behavior. The wavelength λ associated with a particle of momentum p is given by the relation originally proposed by de Broglie:

$$\lambda = h/p \quad (1)$$

where $p = mv$ is the electron momentum (we assume the electron motion is non-relativistic) and h is Planck's constant. The electron momentum p is related to its kinetic energy K by:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m} \quad (2)$$

In this experiment electrons acquire their kinetic energy by being accelerated (essentially from rest) through a potential difference V_a , where they lose potential energy eV_a and gain a corresponding amount of kinetic energy, so that

$$K = eV_a \quad (3)$$

The first experimental observation of the wave behavior of electrons came in an experiment involving diffraction of electrons by a crystalline solid—in principle very similar to this

one. Such diffraction is understood to be a wave phenomenon, and observing it with electrons confirmed their wave aspect.

Bragg Diffraction, Bragg's Law

When a beam of plane electromagnetic waves (such as X-rays) of a single wavelength strikes a crystalline solid, the incident wave interacts with the charged particles in each atom of the solid, causing them to emit individual secondary waves which combine to form an outgoing diffracted beam. Bragg showed that this process could be described by a simple physical picture. One draws sets of equivalent parallel planes through the atoms of the crystal; each distinct set of planes is called a "family" of planes. If the incoming beam makes an angle θ with the planes in a given family, then the outgoing beam makes the same angle with the planes—as if the beam were simply *reflected* from the plane. The outgoing beam is generally rather weak, but when the atoms in adjacent planes emit waves that are in phase, then these waves combine constructively to produce an intense outgoing beam, known as a "Bragg reflection". These intense reflected beams occur when Bragg's Law is satisfied, i.e., when

$$2d \sin \theta = n\lambda \quad (4)$$

where θ is the angle between the incoming beam and a plane in the family, d is the perpendicular distance between adjacent planes of the family, and $n = 1, 2, 3, \dots$ ¹

As we will discover in this experiment, a beam of electrons exhibits similar Bragg reflections to those of a beam of electromagnetic waves, when incident on a crystal.

¹ Here, n is called the *order* of the Bragg reflection.

Preliminary Questions

1. Combine Eqs. 1–3 to obtain an expression for the electron de Broglie wavelength λ as a function of the accelerating voltage V_a .
2. Substitute numerical values for the various quantities and show that if λ is expressed in units of Angstroms (\AA) and V_a is expressed in volts (V), then it follows that:²

² Note: $1 \text{ \AA} = 10^{-10} \text{ m}$.

$$\lambda(\text{\AA}) = \frac{12.26}{\sqrt{V(\text{V})}} \quad (5)$$

3. The smallest spacing between the atoms in a metallic copper crystal is 2.56 Å. What accelerating voltage is needed to produce electrons with de Broglie wavelength equal to that spacing?

Equipment

Teltron electron diffraction tube, filament power supply (AC), high voltage power supply (DC), voltmeter, Vernier calipers

Notes on equipment

The essential interactions of the experiment take place in an evacuated tube, as shown in Figure 1. Electrons are emitted

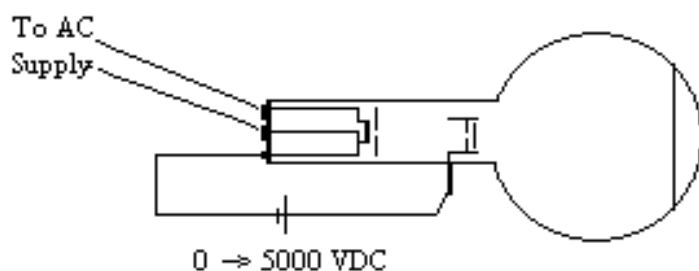


Figure 1: Schematic diagram of electron diffraction tube.

from a heated filament and accelerated by a potential difference V_a . The beam of electrons is incident on a graphite film. The outgoing electrons are detected by means of the light they produce when striking the phosphorescent coating of the tube.

As shown in Fig. 1, the tube contains four terminals for electrical connections, two for the AC voltage (6 V) that serves to heat the filament and two for the DC accelerating voltage V_a . Note that the two larger receptacles on the base of the tube connect to AC terminals of the power supply. The smaller receptacle is to be connected to the negative DC terminal. The positive DC voltage terminal is connected to the plug pointing perpendicular to the tube axis located near the graphite film.

The graphite film is polycrystalline—composed of large number of crystallites (very tiny perfect crystals) randomly

oriented within the material. Referring to Bragg's law (Eq. 4), we note that for a given order n (say $n = 1$) and a given d (for a particular family of planes), there is only one value of the angle θ which satisfies the Bragg condition for a strong reflection. If our sample consisted of a single crystal, we would have to rotate it into position so that the planes would be at the proper angle for the Bragg reflection. However, with our polycrystalline sample, there will always be some crystallites in the proper orientation to give a Bragg reflection at that angle, as illustrated in Figure 2 below.

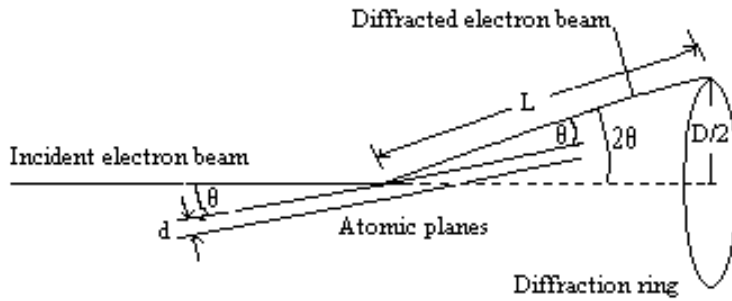


Figure 2: Diffracted electron beam, with ring pattern produced on phosphorescent coating of tube

The figure shows an upward reflection, but downwards or sideways reflections will also occur. Considering all these possibilities, it is easy to see that the reflected electron beams will travel along the sides of a cone whose vertex lies at the point where the electron beam is incident on the sample, and with half-angle equal to 2θ . When these beams strike the spherical end of the tube with the phosphorescent coating, the coating is illuminated in a ring-shaped pattern.

In this experiment, the angle θ is small enough so that the small-angle approximations, $\sin \theta \approx \theta$ and $\sin 2\theta \approx 2\theta$ hold with sufficient accuracy. Using Bragg's law, Eq. 4, we can then relate the measured ring diameter D to the Bragg angle θ and hence to the electron wavelength (see Fig. 1):³

$$\frac{(D/2)}{L} = \sin 2\theta \approx 2\theta \approx 2 \sin \theta = \frac{n\lambda}{d} \quad (6)$$

and hence

$$D \approx \frac{2nL\lambda}{d}. \quad (7)$$

³ Note that for this apparatus, the distance L from the graphite sample to the phosphorescent screen has the value $L = 13.5$ cm.

Procedure

CAUTION

This experiment uses voltages up to 5000 volts DC. Keep hands away from high voltage power supply connections. Ask your instructor to check the circuit before turning on the power supply.

1. Turn on the power supply. The DC voltage should be set at zero. Wait approximately one minute for the AC current to heat the filament and for the filament temperature to stabilize. Then turn up the DC voltage (which is the accelerating voltage V_a for the electron beam) and set it to 4000 volts. Two circular Bragg diffraction rings should be visible on the phosphorescent coating of the tube.⁴
2. Point the bar magnet perpendicular to the straight part of the tube, near the middle of the straight section. How is the Bragg diffraction ring pattern affected? Using the right-hand rule, predict the direction of the magnetic force on the electrons, assuming that they behave as a beam of particles. Compare to the observed deflection of the ring pattern.⁵
3. Adjust the accelerating voltage V_a gradually up and then down. What is the effect on the size of the diffraction rings when V_a is increased?⁶
4. Set the accelerating voltage at 3000 volts. Using the Vernier calipers provided, carefully measure the diameter of each of the two rings. Repeat these measurements for four more values of V_a up to a maximum of 5000 volts, increasing V_a in increments of 500 volts.

⁴ Note: The room should be darkened so that the rings are more visible. Use the lamp provided to assist in taking measurements and recording data.

⁵ Note that in this step, you are observing both the wave nature and particle nature of the electrons with the same apparatus!

⁶ Note that for these observations, it is best to view the rings by looking at the inner surface of the spherical part of the tube, where the phosphorescent coating is located.

Analysis

1. Give a qualitative explanation (in words) for the relationship you observed between the diffraction ring diameter and the accelerating voltage.⁷
2. Based on the theory and the equations discussed for this experiment, derive an expression for the ring diameter D as a function of the accelerating voltage V_a . This expression should be in the form of a power law. What is the theoretically predicted exponent of V_a in this power law?

⁷ Hint: The explanation can be based on the de Broglie hypothesis and a general property of Bragg diffraction. See the above equations and accompanying discussion.

3. Plot on logarithmic axes the experimental values of D vs. V_a for your outer ring data. Does the graph indicate a power-law relationship? Fit your data to find the exponent of V_a and compare it to the theoretical value. Repeat this step for the inner ring data.
4. For the outer ring data, calculate the atomic plane separation d for each value of V_a used. Express your results for d in Angstroms.⁸ Find the mean value of d and the uncertainty in the mean.
5. Repeat the calculations of step 4 for the inner ring data.
6. The crystal structure for graphite—our diffracting material—is hexagonal. The carbon atoms lie in parallel layers and, within each layer, are located at the vertices of regular hexagons. The separation between adjacent atoms is 1.42 \AA . The two families of parallel crystal planes corresponding to the two diffraction rings observed in this experiment are shown in Figure 3 below.

⁸ Hint: Use your data for D and the theory and equations discussed previously. The result of Preliminary Question 2 may be helpful. And note: It turns out that in this experiment we observe only the first-order Bragg reflections, so take $n = 1$ for these calculations.

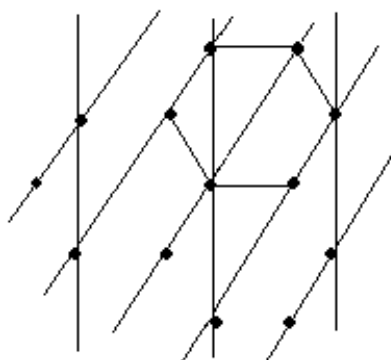


Figure 3: Crystal structure of graphite, showing the two observed families of planes.

For each of the two families of planes shown in the figure, use geometry and trigonometry to calculate a predicted value of d (the perpendicular distance between adjacent planes). Compare these values to the corresponding results for d determined from your measurements, taking the experimental uncertainty into account.

Uncertainty Analysis

The major source of uncertainty in the lab is the width of the diffraction ring, and there is also some uncertainty in the accel-

erating voltage. Use the relative uncertainty in these measurements to estimate the uncertainty in the slope of your log-log plot:

$$\sigma_{slope} = slope \sqrt{\left(\frac{\sigma_{(\log V)}}{\log V}\right)^2 + \left(\frac{\sigma_{(\log D)}}{\log D}\right)^2},$$

where the uncertainty on the logarithmic of a quantity, x , is given by

$$\sigma_{(\log x)} = \frac{\sigma_x}{x \ln(10)}.$$

The experimental value of the aperture or plane spacing should be the same value for all measurements of one ring, so the result should be expressed as an average of the values, with an uncertainty of

$$\sigma_d = \frac{stddev(d)}{\sqrt{\text{number of measurements}}}.$$

Reference

- French and Taylor, An Introduction to Quantum Physics, pp. 72-78
- Thornton and Rex, Modern Physics 3rd edition, pp. 171-173
- Tipler and Llewellyn, Modern Physics 5th edition, pp. 185-192