would need to be less than one inch long (Ex. 55) to produce electrons with the same speed!

4. The Coordinates of an Event

The inertial reference frame is to a student of physics what the north-south east-west grid of lines in a township is to a surveyor. The surveyor is concerned with position in space. The student of physics is concerned with location of an event in space and in *time*. The Daytime and Nighttime surveyors *could* have dispensed with north-south and east-west coordinates and simply measured the *distance* between any two gates; but at the start they did not even know there was any such quantity as "distance." In the same way in this chapter we could have gone about locating events in spacetime solely by measuring the *intervals* between one event and another, without any regard for "space" and "time" coordinates individually.† However, we have to start as physics did before 1905, without benefit of any concept of interval. This concept will force itself upon our attention as the concept of distance forced itself upon the surveyors. The two men measured north-south and east-west coordinates in two different coordinate systems. Only later did they see the connection ("invariance of the distance") between the very different numbers in their notebooks. Similarly, we will begin with space and time coordinates of events in the laboratory reference frame, and space and time coordinates of the same events in the rocket reference frame. Then there will be a firm basis for concluding that the interval between two events as determined from laboratory numbers is identical with the interval between the same two events as calculated from the very different rocket readings ("invariance of the interval").

Why use coordinates?

The fundamental concept in surveying is a *place*. The fundamental concept in physics is an *event*. An event is specified not only by a place but also by a time of happening. Some examples of events are: emission of particles or flashes of light (explosions), reflection or absorption of particles or light flashes, collisions, and near-collisions called *coincidences*.

Event defined

How can one determine the place and time at which an event occurs in a given inertial reference frame? Think of constructing a frame by assembling meter sticks into a cubical latticework similar to the "jungle gym" seen on playgrounds (Fig. 9). At every intersection of this latticework fix a clock. These clocks can be constructed in any way, but are *calibrated* in meters of light-travel time. In Section 1 we discussed how to obtain such a calibration by bouncing a flash of light back and forth between two mirrors one-half meter apart. This mirror clock is said to "tick" each time the light flash arrives back at the first mirror. Between ticks the light flash travels a round-trip distance of 1 meter: Call the unit of time between ticks *1 meter of light-travel time*, or more simply, *1 meter of time*. The speed of light in conventional units has the measured value $c = 2.997925 \times 10^8$ meters per second. Light will travel 1

Latticework of clocks

†Such a treatment is presented by Robert F. Marzke and John A. Wheeler in *Gravitation and Relativity*, edited by H.-Y. Chiu and W. F. Hoffmann, (W. A. Benjamin, New York, 1964).

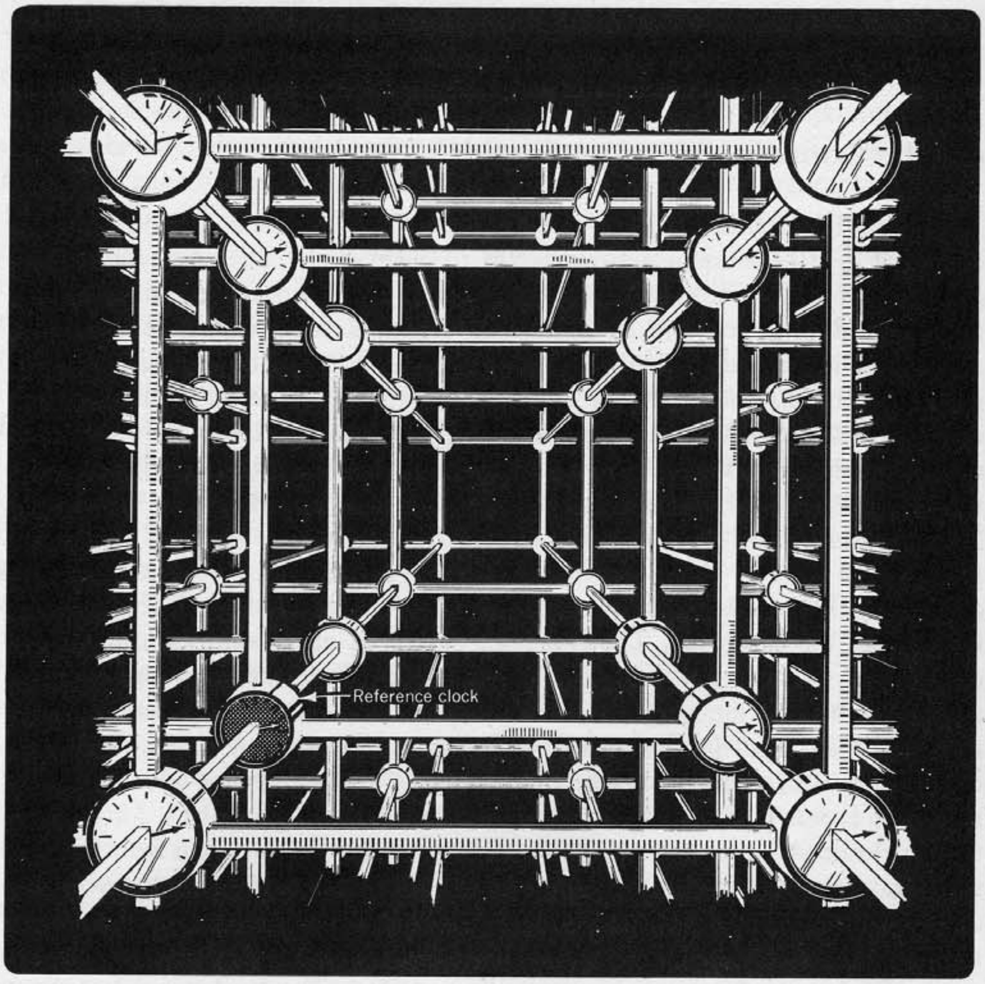


Fig. 9. Latticework of meter sticks and clocks.

meter in the time, 1 meter/c = 3.335640×10^{-9} seconds. Hence *1 meter of light-travel time is equal to 3.335640×10^{-9} seconds*—about 3.3 *nanoseconds* in the terminology of high-speed electronic circuits! We assume that *every* clock in the latticework, whatever its construction, has been calibrated in meters of light-travel time.

Synchronizing clocks in lattice

How are the different clocks in the lattice to be *synchronized* with one another? As follows: Pick one of the clocks in the lattice as the standard of time and take it to be the *origin* of an x, y, z coordinate system. Start this reference clock with its pointer at $t = 0$. At this instant let it send out a flash of light that spreads in all directions. Call this flash of light the *reference flash*. When the reference flash gets to a clock 5 meters away, we want that clock to read 5 meters of light-travel time. So an assistant *sets* that clock to 5 meters of time long before the experiment begins, *holds* it at 5 meters, and *releases* it only when the reference flash arrives. When assistants at all the clocks in the lattice have followed this procedure (each setting his clock to a time in meters equal to his own distance from the reference clock and starting it when the light flash arrives), the clocks in the lattice are said to be *synchronized*.

There are other possible ways to synchronize clocks. For example, an extra portable clock could be set to the reference clock at the origin and then carried around the lattice in order to set the rest of the clocks. However, this procedure involves a *moving* clock. We will see later that a moving clock runs at a different rate as measured by clocks in the lattice than do clocks that remain at rest in the lattice. The portable clock will not even agree with the first one when it is brought back next to it! (Clock paradox, Ex. 27). However, if we use a moving clock that travels at a speed that is a *very small fraction* of the speed of light, it is only slightly in error, and this second method of synchronization will give very nearly the same result as the first—and standard—method. Moreover the error can be made as small as desired by carrying the portable clock about sufficiently slowly.

Latticework used to measure the four coordinates of event

The latticework of synchronized clocks can be used to determine the location and time at which any given event occurs. The *location* of the event will be taken to be the position of the clock nearest the event. The *time* of the event will be taken to be the time recorded on the lattice clock nearest the event. The *coordinates* of an event will then consist of four numbers: three numbers that specify the *spatial position* of the clock nearest the event and the *time* (in meters) when the event occured as recorded by that clock. The clocks, if they are installed by a foresighted experimenter, will be *recording* clocks. Each will be able to detect the occurrence of an event (passage of a light flash or particle). Each will punch onto a card the nature of the event, the time of the event and the location of the clock. The cards can then be collected from all the clocks and later analyzed, perhaps by automatic equipment.

Lattice spacing depends on scale of physics under study

Why a lattice built of rods that are *one* meter long? When a clock in this lattice punches out a card, one will not know whether the event so recorded is 0.4 meters to the left of the clock, for instance, or 0.2 meters to the right. The location of the event will be uncertain by some substantial fraction of a meter. The time of the event will also be unknown within some appreciable fraction of a meter of light-travel time. This accuracy, however, is quite adequate for observing the passage of a rocket. It is extravagantly good for measurements on planetary orbits—it would even be reasonable to increase the lattice spacing from 1 meter to hundreds of meters. Neither 100 meters nor 1 meter is a lattice spacing suitable for studying the tracks of particles from a high-energy accelerator. There a centimeter or a millimeter would be more appropriate. The location and time of an event can be determined to whatever accuracy is desired by constructing a lattice with sufficiently small spacing.

Observer defined

In relativity we often speak about "the observer." Where is this observer? At one place or all over the place? *The word "observer" is a shorthand way of speaking about the whole collection of recording clocks* associated with one inertial frame of reference. No one real observer could easily do what we ask of the "ideal observer" in our analysis of relativity. So it is best to think of the observer as the man who goes around picking up the punched cards turned out by all the recording clocks in his employ. This is the sophisticated sense in which we will hereafter be using the phrase "the observer finds such and such."

Clock records reveal motion of particle through lattice

The clocks reveal the motion of a particle through the lattice: each clock that the particle passes punches out the time of passage as well as the space coordinate of this event. How can the path of the particle be described in terms of numbers? By recording the coordinates of these events along this path. Differences between the coordinates of successive events reveal the velocity of the particle. The conventional units for speed, v, are meters per second. However, when time is measured in meters of light-travel time, then speed is expressed in meters of distance covered per meter of time. To avoid confusion, speed expressed in meters per meter will be represented by the Greek letter beta, β. A flash of light moves one meter of distance in one meter of light-travel time, or, $\beta_{\text{light}} = 1$. The speed of other particles in meters per meter represents a fraction of the speed of light. In other words $\beta = v/c$. Here and hereafter c represents the speed of light.

Verifying that lattice furnishes inertial frame

From the motion of test particles through the latticework of clocks—or rather from the records of coincidences punched out by the recording clocks—we can determine whether the latticework constitutes an inertial reference frame. *If* the records show (a) that—within some specified accuracy—a test particle moves consecutively past clocks that lie in a straight line, (b) that the speed β of the test particle calculated from the same records is constant—again, within some specified accuracy—and, (c) that the same results are true for as many test-particle paths as the most industrious observer cares to trace throughout the given region of space and time, *then* the lattice constitutes an inertial reference frame throughout that region of spacetime.

Laboratory and rocket frames; x axes coincide

Once again we have described the motion of test particles with respect to a particular reference frame in order to determine whether that frame is inertial. The same test particles and—if they collide—the same collisions may be described with reference to one inertial frame as well as another. Let two reference frames be two different latticeworks of meter sticks and clocks, one moving uniformly relative to the other, and in such a way that their x axes coincide. Call one of these frames the *laboratory frame* and the other—moving in the positive x direction relative to the laboratory frame—the *rocket frame* (Figs. 10 and 11). The rocket is *unpowered* and coasts along with constant velocity relative to the laboratory. Let the rocket and laboratory latticeworks be *overlapping* in the sense that there is a region of spacetime common to both frames (as described in Section 3 and Fig. 8). Test particles move through this common region of spacetime. From the motion of these test particles as recorded by his own clocks, an observer in each frame verifies that his frame is *inertial.*

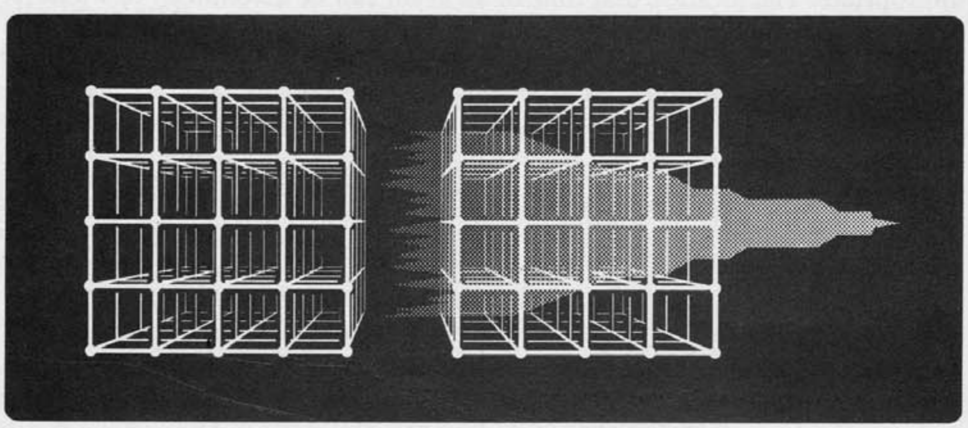


Fig. 10. Laboratory and rocket frames. The two latticeworks intermeshed a second ago.

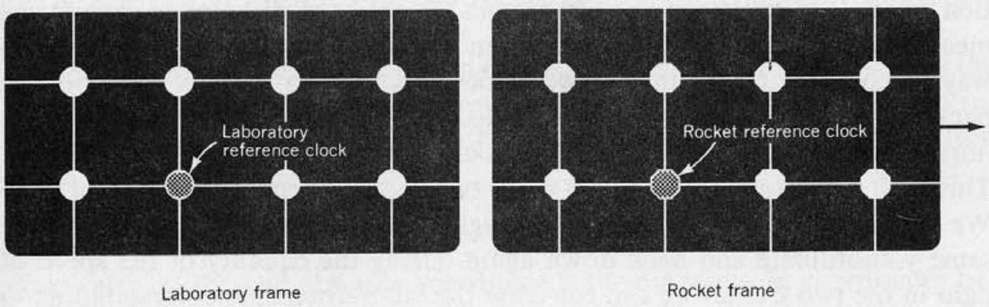


Fig. 11. Laboratory and rocket frames, further schematized from Fig. 10. The central reference clock of each frame is shaded.

Laboratory and rocket observers record single event

A firecracker explodes. The explosion is recorded by the clock in the laboratory lattice nearest to the explosion. It is recorded also by the clock in the rocket lattice nearest to the explosion. How do the coordinates of the recording laboratory clock compare with the coordinates of the recording rocket clock? One result can be derived immediately from the principle of relativity: the recording laboratory and rocket clocks will have the same y coordinates. To show this, let the recording rocket clock carry a wet paint brush that makes marks on the laboratory lattice as it moves past. Figure 12 shows this for the special case, $y = 1$ meter. The marks on the laboratory lattice serve to measure the laboratory y coordinate of the $y = 1$ rocket clock. These paint marks appear *on* the $y = 1$ laboratory clocks rather than *above* them or *below* them. For suppose that the paint marks appear on the lattice rods *below* the $y = 1$ laboratory clocks: Then *both* observers will agree that the $y = 1$ rocket clocks passed "inside" the $y = 1$ laboratory clocks. Permanent paint marks would verify this for all to see. Similarly, if the paint marks appear on the lattice rods *above* the $y = 1$ laboratory clocks, both observers will agree that the $y = 1$ rocket clocks passed "outside" the $y = 1$ laboratory clocks. In either case there would be a way to distinguish experimentally between the two frames. But no one has been able to distinguish between these two frames using any *other* experiment. The principle of relativity embodies the assumption that any such experimental distinction between inertial reference frames is impossible. Therefore we assume that no one could distinguish between the two frames using *this* experiment. It follows that the y coordinate of *any* event—such as the explosion that began this paragraph—will have the same y coordinate in the rocket frame as in the laboratory frame.

y coordinate of event is same in lab and rocket frames

By a similar argument the z coordinate of an event is the same in the rocket frame as in the laboratory frame. Notice that both the y coordinate and the z coordinate of an event are measured in a direction *perpendicular* to the direc-

z coordinate of event is same in lab and rocket frames

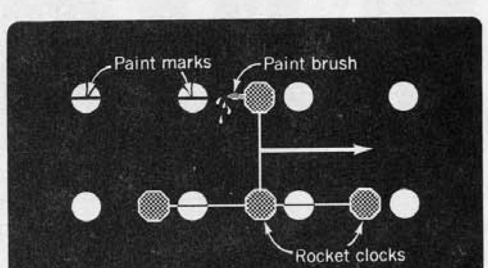


Fig. 12. Demonstration that y coordinate of an event is the same in laboratory and rocket frames.

tion of relative motion of the two frames. Equality of distance in each frame measured perpendicular to the direction of relative motion gives us a clean way to compare clocks in the two latticeworks. Let a flash of light bounce back and forth between a mirror mounted on the rocket reference clock and a mirror mounted on the $y = 1$ rocket clock directly above the reference clock. This flash will return to the origin every two meters of rocket light-travel time. We can trace the path of this flash of light in the laboratory frame up to the same y coordinate and back down again. Using the equality of the speed of light in the two frames we can calculate the laboratory time corresponding to the 2-meter round-trip time in the rocket frame. In the next section this study will lead to a demonstration of the invariance of the interval.

5. Invariance of the Interval

Looking for measure of separation AB that has same value in all inertial frames

Distance between two town gates is calculated from the difference between the x coordinates of the two gates and the difference between the y coordinates. How does one find the analogous physical quantity, the spacetime interval between two events? And between what two events shall this interval be evaluated?

Let *event A* be the *emission* of a flash of light. Let *event B* be the *reception* of this flash after its reflection from another object. All that matters in the end is the pair of events. Neither the light nor the object that reflects it is of any direct interest. Nevertheless, an analysis of the track of this pulse through spacetime reveals quickly and simply a quantity (the interval) that is associated with the two events and that has the same value in all inertial reference frames.

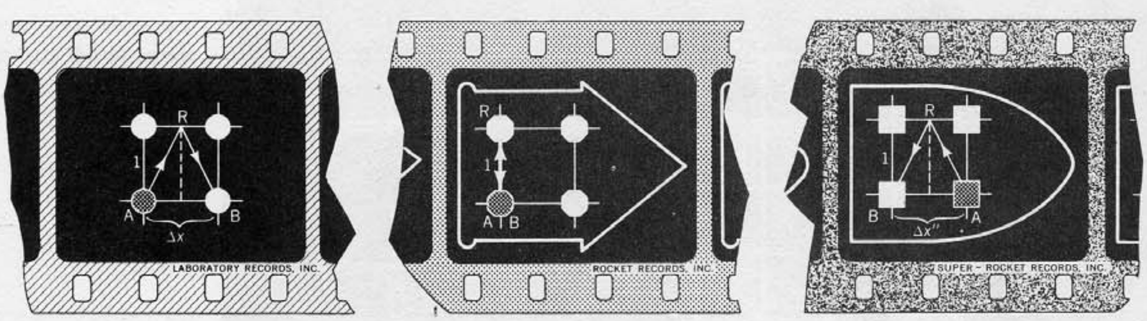
Event A: flash emission
Event B: flash reception

Event A: A spark plug fires. A flash of light flies up to reflector R in Fig. 13. Then the flash wings down. *Event B:* The flash is recorded. Now for the details (Figure 13).

The spark plug fires in the laboratory frame at the zero of time and at the origin of the x, y, z coordinate system (crosshatched). The rocket passes by with such timing that it records the spark as taking place also at *its* origin (likewise crosshatched) and at *its* zero of time. So much for the coordinates of the event of emission:

Details: lab and rocket coordinates of events A and B

$$x_{\text{emission}} = 0, \quad y_{\text{emission}} = 0, \quad t_{\text{emission}} = 0 \quad \text{(laboratory frame)}$$
$$x'_{\text{emission}} = 0, \quad y'_{\text{emission}} = 0, \quad t'_{\text{emission}} = 0 \quad \text{(rocket frame)}$$



A. Light path as observed in laboratory frame
B. Light path as observed in rocket frame
C. Light path as observed in super-rocket frame

Fig. 13. Emission and reflection of the reference flash, and its reception at origin of rocket frame.