Test particle defined

"Test particle." How small must a particle be to qualify as a test particle? It must have so little mass that, within some specified accuracy, its presence will not affect the motion of other nearby particles. In terms of Newtonian mechanics the gravitational attraction of the test particle for other particles must be negligible within the accuracy specified. As an example, consider a particle of mass 10 kilograms. A second and less massive particle placed one-tenth meter from it and initially at rest will, in less than three minutes, undergo a displacement of 10^{-3} meters. Thus the 10-kilogram object is not—in this sense—a test particle. A test particle *responds* to gravitational forces but it does not itself *produce* any significant gravitational force.

Inertial frame is definable because all substances fall with same acceleration

It would be impossible to define an inertial reference frame if it were not for a remarkable feature of nature. Particles of different size, shape, and material in the same location all fall with the *same acceleration* toward the earth. If this were not so, an observer inside a falling space ship would notice a relative acceleration among different particles even when they are close together; at least some of the particles initially at rest would not remain at rest; that is, the space ship would not be an inertial reference frame according to the definition. How sure are we that particles in the *same* location but of *different* substances all fall toward the earth with the same acceleration? According to legend Galileo dropped balls made of different materials from the Leaning Tower of Pisa in order to verify this assumption.† In 1922 Baron Roland von Eötvös checked to an accuracy of five parts in 10^9 that the earth imparts the same acceleration to wood and to platinum. More recently Robert H. Dicke has pointed out that the sun is more suitable than the earth as source for the gravitational acceleration that one will measure (see Ex. 35). The alternation in direction of the sun's pull every 12 hours lends itself to fantastic amplification by resonance. Cylinders of aluminum and gold experience accelerations due to the sun (0.59×10^{-2} meters per second per second) that are the same to three parts in a hundred thousand million (3 in 10^{11}), according to R. H. Dicke and Peter G. Roll.‡ This is one of the most sensitive checks of a fundamental physical principle in all of physics: the identity of the acceleration produced by gravity in every kind of test particle.

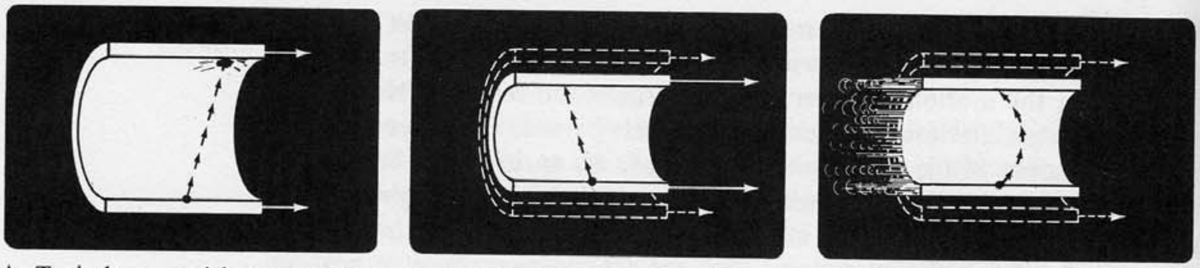
It follows from this principle that a particle made of *any* material can be used as a test particle to determine whether a given reference frame is inertial. A reference frame that is inertial for one kind of test particle will be inertial for all kinds of test particles.

3. The Principle of Relativity

We describe the motion of test particles with reference to a particular reference frame in order to determine whether that frame is inertial. The same test particles and—if they collide—the same collisions may be described with reference to more than one inertial frame. The one reference frame might be carried by a space ship built like a hollow cylinder (Fig. 8,A), the other by a second craft of similar construction just enough smaller to zoom through in-

†On the question whether Galileo actually performed this experiment, see *Physics the Pioneer Science* by Lloyd W. Taylor, (Dover Publications, New York, 1959), Vol. 1, p. 25.

‡See the chapter on experimental relativity by Dicke in *Relativity, Groups, and Topology*, edited by C. and B. DeWitt, (Gordon and Breach, New York, 1964), pp. 173–177 or the book by Dicke *The Theoretical Significance of Experimental Relativity* (Gordon and Breach, New York, 1964).



A. Typical test particle as seen from one inertial reference frame

B. Same test particle as seen from a second inertial reference frame moving with respect to the first one

C. Same test particle as seen from an accelerated—and therefore *non-inertial*—reference frame

Fig. 8. Inertial reference frames versus accelerated reference frame.

side as it catches up with the first and passes it (Fig. 8,B). There is a region of spacetime common to the interior of both vehicles during the time of passing. Flying across this region in one direction or another are numerous test particles. Every track will be straight as plotted with respect to the coordinates of one reference frame—and also with respect to the other frame—because both are *inertial* frames. This straightness in both frames is possible only because *one inertial reference frame has uniform velocity relative to any second and overlapping inertial reference frame.* In contrast, if the second rocket ship is powered so that it accelerates as it passes through the first ship (Fig. 8,C), test particles will follow curved paths—as observed from this second rocket. If the curvature of these paths is detectable with given equipment, then such an accelerated reference frame is a noninertial frame.

Overlapping inertial frames move with uniform relative velocity

In each of two *inertial* frames in uniform relative motion, every test particle that is in motion continues that motion without change in speed or in direction, even though the direction and speed of a given particle will not look the same in both frames. Indeed, we have *defined* the inertial frame in such a way that the following law of mechanics (Newton's first law) is true in every inertial reference frame: "A free particle at rest remains at rest, and a free particle in motion continues that motion without change in speed or in direction." There are additional laws of mechanics. Each of these laws also holds true—according to experiment—in every inertial reference frame.

Do other laws of physics maintain their validity in every inertial frame? In designing electrical circuits for a jet plane must an electrical engineer use different circuit laws because the plane will be moving? Must different electromagnetic laws of radiation be used in designing a radio transmitter for a space probe because of the motion of the probe? If a small proton accelerator together with its targets and particle-detecting equipment is mounted on a railway flatcar, will the interpretation of collision experiments with protons require the use of different laws when the flatcar is moving uniformly than when the flatcar is at rest? As far as we know the answer to these three questions and others like them is "No." In spite of the most diligent search no one has ever found any violation of the following principle:

Relativity: laws of physics are same in every inertial frame

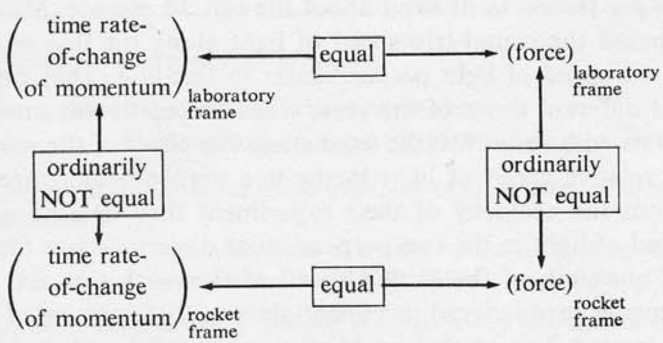
All the laws of physics are the same in every inertial reference frame.

We will call this statement the *principle of relativity*. The principle of relativity says that once the laws of physics have been derived in one inertial reference frame, they can be applied without modification in any other inertial reference frame. Both the *form* of the laws of physics and the *numerical values of the physical constants* that these laws contain are the same in every inertial reference frame. All inertial frames are equivalent in terms of *every* law of physics. Expressed in negative terms, the principle of relativity says that *the laws of physics cannot provide a way to distinguish one inertial frame from another*—any more than the surveyor's tape and level give a means to tell North Star north from magnetic north!

What principle of relativity does NOT say!

Notice what the principle of relativity does *not* say. It does not say that the time between events A and B will appear the same when measured from two different inertial reference frames. Neither does it say that spatial separation between the two events will be the same in the two frames. Ordinarily neither times nor distances will be the same in the two frames—any more than the northward and eastward components of the separation of gates A and B as read by the Daytime surveyor are identical to those recorded by the Nighttime surveyor. In consequence the momentum of a given particle will have a value in one frame that is different from its value in a second frame. Even the time rate-of-change of momentum will ordinarily differ between the two frames. And so will the force. Thus, when studying the motion of a charged particle, two observers in relative motion will not necessarily find the same values for the electric field or the magnetic field acting upon this charged particle. The total force, produced by the electric and magnetic fields together, will differ between the one inertial reference frame and the other.

The physics that is so different between one frame and the other is nevertheless the same in the two frames! Physical quantities differ in *value* between the two frames but fulfill identical *laws*. The time rate-of-change of momentum in one frame is equal to the total force as measured in that frame (Newton's second law). The time rate-of-change of momentum in the second frame is equal to the total force as measured in that second frame:



Not only the laws of mechanics but also the laws of electromagnetism and all other laws of physics hold true as well in one inertial reference frame as in any other inertial reference frame. This is what it means to say that "the laws of physics cannot provide a way to distinguish one inertial reference frame from another."

The laws of electromagnetism hold true as well in one inertial reference frame as in any other inertial reference frame. The numerical value of the speed of light, $c = 2.997925 \times 10^8$ meters per second, is one of the constants which appears in the laws of electromagnetism. According to the principle of relativity this experimental value must be the same in each of two inertial reference frames in uniform relative motion. Has experiment shown this to be true? The answer is yes, but the experiments to date are much less sensitive than they should be for so important a question. For the moment let us pull in our horns and concentrate on a simpler question which *can* be answered definitively. The laws of electromagnetism contain no expressions that depend upon direction. Therefore one expects to find the round-trip speed of a flash of light to be the same whether the flash travels north-south or east-west: the speed of light is *isotropic*. But now let these same flashes of light be observed from a uniformly moving rocket. With respect to this rocket, will the round-trip speed of light not be different for light moving out and back along different lines? The principle of relativity says *no*: the speed of light, which is isotropic in one inertial frame, is also isotropic in all other inertial frames that share the same spacetime region.

Michelson-Morley experiment: speed of light is isotropic in all inertial frames

How strange this result is! We know that the speed of sound in air is the same in all directions if the air is still. But let a stiff wind be blowing—or, to get the same result, move through the still air in an automobile. Then, with respect to the automobile, the "downstream speed of sound" is greater than the "upstream speed of sound." And a simple calculation shows that both these speeds are different from the speed of sound measured across the wind. The round-trip speed of sound measured with respect to the automobile will be different in different directions. The same result is true for every other form of wave motion we know about—except that of light! How can we be so sure that this result is *not* true for experiments with light? Our assurance is based on a series of refined experiments beginning with the classic experiment of A. A. Michelson and E. W. Morley performed after 1880.† They used the earth itself as a moving reference frame. (The earth is effectively an inertial frame for local experiments with light—see Ex. 31.) The earth moves at a speed of about 30 kilometers per second in its orbit about the sun. In essence, Michelson and Morley compared the round-trip speed of light along the line of the earth's motion with the speed of light perpendicular to this line. They repeated this experiment at different times of the year, when the earth was moving in different directions with respect to the fixed stars. No effect of the motion of the earth on the relative speed of light in the two perpendicular directions was observed. From the accuracy of their experiment they determined that the measured speed of light in the two perpendicular directions was the same to a sensitivity of one-sixth of the orbital speed of the earth (see Ex. 33). More recent experiments have reduced this uncertainty to three percent of the orbital speed of the earth.‡ The Michelson-Morley experiment and its modern improvements tell us that in every inertial frame the round-trip speed of light is the same in every direction—the speed of light is *isotropic* in both laboratory and rocket frames as predicted by the principle of relativity. But the principle of relativity says more than this. Not only must the speed of light be isotropic in the laboratory frame—and also isotropic in the rocket frame—but also, if

†A. A. Michelson and E. W. Morley, American Journal of Science, **34,** 333 (1887).

‡T. S. Jaseja, A. Javan, J. Murray, and C. H. Townes, Physical Review, **133,** A1221 (1964).

Table 4. Modern tests to answer the question, "Does the round-trip speed of light differ between one reference frame and another?"

TWO REFERENCE FRAMES

ONE REFERENCE FRAME

The earth moving in one direction around the sun in, say, January.

ANOTHER REFERENCE FRAME

The earth moving in the opposite direction (with respect to the fixed stars) in July.

EXPERIMENTAL RESULTS

RESULT OF THE MICHELSON-MORLEY EXPERIMENTS

Original experiment

Observers in neither frame (may be the *same* observer on earth who repeats the experiment after waiting six months) can detect differences in the round-trip speed of light in any two perpendicular directions greater than one-sixth of the speed of the earth in its orbit.

The more modern experiment

Observers in neither frame can detect differences in the round-trip speed of light in any two perpendicular directions greater than three percent of the speed of the earth in its orbit.

RESULT OF THE KENNEDY-THORNDIKE EXPERIMENT

The round-trip speed of light is the same in one of the seasonal reference frames defined above as in the other reference frame with a sensitivity of about two meters per second.

INTERPRETATION OF THE EXPERIMENTS

THE MORE MODERN MICHELSON-MORLEY EXPERIMENT

The speed of the earth in its orbit about the sun is

30 kilometers per second = 1/10,000 of the speed of light

Thus the *difference* of the round-trip speed of light measured in two perpendicular directions is

less than 3/100 of 1/10,000 of the speed of light
which is less than 3/1,000,000 of the speed of light

Therefore the *principle of relativity* is supported by this modern experiment with a sensitivity of

three parts in a million

THE KENNEDY-THORNDIKE EXPERIMENT

The *difference* of the round-trip speed of light as measured in the two frames is
less than about 2 meters per second
which is less than 1/100,000,000 of the speed of light

Therefore the *principle of relativity* is supported by this experiment with a sensitivity of

one part in a hundred million

Kennedy-Thorndike experiment: speed of light has same numerical value in all inertial frames

this principle is correct, the *numerical value* of this isotropic speed, $c = 2.997925 \times 10^8$ meters per second, must be the same in the rocket frame as in the laboratory frame. Can this prediction also be verified by experiment? This verification was carried out by R. J. Kennedy and E. M. Thorndike about 50 years after Michelson and Morley did their experiment.† Like Michelson and Morley, Kennedy and Thorndike used the earth as a moving reference frame. They tried to detect any variation in the magnitude of the round-trip speed of light as the earth moved in different directions around the sun at different times of the year. From the accuracy of their negative results one can conclude that there is no difference as great as about two meters per second in the round-trip speed of light as between two reference frames with a relative velocity of 60 kilometers per second (twice the speed of the earth in its orbit; see Ex. 34). In the Kennedy-Thorndike experiment the standard of length is the interferometer base itself, a single block of fused quartz kept in a vacuum at a temperature constant to about a thousandth of a degree. The standard of time is provided by the characteristic vibration period associated with a particular green spectral line of a mercury atom. Keeping conditions constant for months constituted the most important single difficulty—and difference—of this Pasadena, California experiment as contrasted to the Cleveland, Ohio Michelson-Morley experiment, where the relevant comparisons (one direction as against another) could be made in the course of a single day. Table 4 summarizes the conclusions of the Michelson-Morley and the Kennedy-Thorndike experiments.

Although the sensitivity of neither of these experiments is as great as that of the Eötvös-Dicke experiment (three parts in a hundred thousand million), the results are nonetheless striking experimental support for the principle of relativity. Happily, there are plans to improve the sensitivity of the Kennedy-Thorndike experiment.‡ This improvement in sensitivity is important. The measurement of time in meters of light-travel time has meaning only if light travels one meter in the same time in all frames. The equality of the speed of light in rocket and laboratory frames provides a simple way (Section 5) to compare clocks between the two frames. This comparison depends for its validity on the null result of the Kennedy-Thorndike experiment.

Structure of spacetime makes LINAC cost $300,000,000

In 1905 the principle of relativity was a shocking heresy, which offended the intuition and common-sense way of looking at nature of most physicists. It has taken a long time to become accustomed to the apparently absurd idea that one particular speed has the same value measured in two overlapping inertial frames in relative motion. The principle of relativity is used every day in many fields of physics where it is continually under severe tests. For example, the Stanford Linear Electron Accelerator (estimated cost: $300,000,000) has to be two miles long to push electrons up to a speed that is almost the speed of light (the difference from the speed of light is only 8 parts in 10^{11}). If the pre-Einstein Newtonian laws of mechanics were correct, then the accelerator

†R. J. Kennedy and E. M. Thorndike, Physical Review, **42,** 400 (1932).

‡See Jaseja, Javan, Murray, and Townes, Physical Review, **133,** A1221 (1964). For a careful analysis of the experimental foundations of special relativity see H. P. Robertson, "Postulate versus Observation in the Special Theory of Relativity," Reviews of Modern Physics, **21,** 378 (1949).

would need to be less than one inch long (Ex. 55) to produce electrons with the same speed!

4. The Coordinates of an Event

The inertial reference frame is to a student of physics what the north-south east-west grid of lines in a township is to a surveyor. The surveyor is concerned with position in space. The student of physics is concerned with location of an event in space and in *time*. The Daytime and Nighttime surveyors *could* have dispensed with north-south and east-west coordinates and simply measured the *distance* between any two gates; but at the start they did not even know there was any such quantity as "distance." In the same way in this chapter we could have gone about locating events in spacetime solely by measuring the *intervals* between one event and another, without any regard for "space" and "time" coordinates individually.† However, we have to start as physics did before 1905, without benefit of any concept of interval. This concept will force itself upon our attention as the concept of distance forced itself upon the surveyors. The two men measured north-south and east-west coordinates in two different coordinate systems. Only later did they see the connection ("invariance of the distance") between the very different numbers in their notebooks. Similarly, we will begin with space and time coordinates of events in the laboratory reference frame, and space and time coordinates of the same events in the rocket reference frame. Then there will be a firm basis for concluding that the interval between two events as determined from laboratory numbers is identical with the interval between the same two events as calculated from the very different rocket readings ("invariance of the interval").

Why use coordinates?

The fundamental concept in surveying is a *place*. The fundamental concept in physics is an *event*. An event is specified not only by a place but also by a time of happening. Some examples of events are: emission of particles or flashes of light (explosions), reflection or absorption of particles or light flashes, collisions, and near-collisions called *coincidences*.

Event defined

How can one determine the place and time at which an event occurs in a given inertial reference frame? Think of constructing a frame by assembling meter sticks into a cubical latticework similar to the "jungle gym" seen on playgrounds (Fig. 9). At every intersection of this latticework fix a clock. These clocks can be constructed in any way, but are *calibrated* in meters of light-travel time. In Section 1 we discussed how to obtain such a calibration by bouncing a flash of light back and forth between two mirrors one-half meter apart. This mirror clock is said to "tick" each time the light flash arrives back at the first mirror. Between ticks the light flash travels a round-trip distance of 1 meter: Call the unit of time between ticks *1 meter of light-travel time*, or more simply, *1 meter of time*. The speed of light in conventional units has the measured value $c = 2.997925 \times 10^8$ meters per second. Light will travel 1

Latticework of clocks

†Such a treatment is presented by Robert F. Marzke and John A. Wheeler in *Gravitation and Relativity*, edited by H.-Y. Chiu and W. F. Hoffmann, (W. A. Benjamin, New York, 1964).