
SPACETIME PHYSICS

TAYLOR/WHEELER

A Brief, Readable Exposition
of Modern RELATIVITY THEORY

Illustrated and Amplified
by a Wealth of
PROBLEMS, PUZZLES, and PARADOXES
and Their Detailed Solutions

Selected Physical Constants

Speed of light in a vacuum	$c = 2.997925 \times \begin{cases} 10^8 \text{ meters/second} \\ 10^{10} \text{ centimeters/second} \end{cases}$ $c = \begin{cases} 1 \text{ meter of distance/meter of light-travel time} \\ 1 \text{ centimeter of distance/centimeter of light-travel time} \end{cases}$
Gravitational constant	$G = 6.670 \times \begin{cases} 10^{-11} \text{ meter}^3/(\text{kilogram-second}^2) \\ 10^{-8} \text{ centimeter}^3/(\text{gram-second}^2) \end{cases}$
Planck constant	$h = 6.6256 \times \begin{cases} 10^{-34} \text{ kilogram-meter}^2/\text{second} \\ 10^{-27} \text{ gram-centimeter}^2/\text{second} \end{cases}$
Quantum of angular momentum	$\hbar = 1.0545 \times \begin{cases} 10^{-34} \text{ kilogram-meter}^2/\text{second} \\ 10^{-27} \text{ gram-centimeter}^2/\text{second} \end{cases}$
Boltzmann constant	$k = 1.38054 \times \begin{cases} 10^{-23} \text{ joule/degree Kelvin} \\ 10^{-16} \text{ erg/degree Kelvin} \end{cases}$
Elementary charge	$e = \begin{cases} 1.60210 \times 10^{-19} \text{ coulombs} \\ 4.80298 \times 10^{-10} \text{ esu or (gram centimeter}^3/\text{second}^2)^{1/2} \end{cases}$
Electron rest mass	$m_e = 9.1091 \times \begin{cases} 10^{-31} \text{ kilogram} \\ 10^{-28} \text{ gram} \end{cases}$
Electron rest energy	$m_e c^2 = 8.1869 \times \begin{cases} 10^{-14} \text{ joules} \\ 10^{-7} \text{ ergs} \end{cases}$ $= 0.510984 \text{ MeV}$
Proton rest mass	$m_p = 1.67252 \times \begin{cases} 10^{-27} \text{ kilogram} \\ 10^{-24} \text{ gram} \end{cases}$
Proton rest energy	$m_p c^2 = 1.503186 \times \begin{cases} 10^{-10} \text{ joules} \\ 10^{-8} \text{ ergs} \end{cases}$ $= 938.232 \text{ MeV}$
Mass of the earth	$M_{\oplus} = 5.977 \times \begin{cases} 10^{24} \text{ kilograms} \\ 10^{27} \text{ grams} \end{cases}$
Radius of a sphere having the same volume as the earth	$R_{\oplus} = 6.371 \times \begin{cases} 10^6 \text{ meters} \\ 10^8 \text{ centimeters} \end{cases}$
Mean distance of the earth from the sun = "astronomical unit"	$AU = 1.495985 \times \begin{cases} 10^{11} \text{ meters} \\ 10^{13} \text{ centimeters} \end{cases}$
Mean speed of the earth in its orbit about the sun	$v_e = 29.8 \text{ kilometers/second}$
Mean distance of the moon from the earth	$3.84 \times \begin{cases} 10^8 \text{ meters} \\ 10^{10} \text{ centimeters} \end{cases}$
Mass of the sun	$M_{\odot} = 1.989 \times \begin{cases} 10^{30} \text{ kilograms} \\ 10^{33} \text{ grams} \end{cases}$
Mean radius of the sun	$R_{\odot} = 6.9598 \times \begin{cases} 10^8 \text{ meters} \\ 10^{10} \text{ centimeters} \end{cases}$

Conversion Factors

1 second = $2.997925 \times \begin{cases} 10^8 \text{ meters} \\ 10^{10} \text{ centimeters} \end{cases}$ of light-travel time
1 meter of light-travel time = 3.335640×10^{-9} second
1 centimeter of light-travel time = 3.335640×10^{-11} second
1 year = 3.156×10^7 seconds = $9.460 \times \begin{cases} 10^{16} \text{ meters} \\ 10^{17} \text{ centimeters} \end{cases}$ of light-travel time
1 kilometer = 0.6214 mile
1 electron-volt = 1.602×10^{-19} joule = 1.602×10^{-12} erg

Edwin F. Taylor

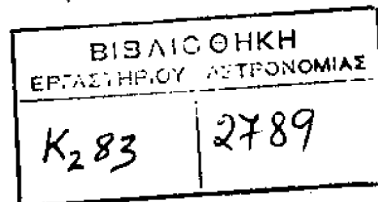
Massachusetts Institute of Technology

John Archibald Wheeler

Princeton University

Spacetime Physics

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The Geometry of Spacetime

1

1. Parable of the Surveyors

Once upon a time there was a Daytime surveyor who measured off the king's lands. He took his directions of north and east from a magnetic compass needle. Eastward directions from the center of the town square he measured in meters (x in meters). Northward directions were sacred and were measured in a different unit, in miles (y in miles). His records were complete and accurate and were often consulted by the Daytimers. (See Fig. 1.)

Daytime surveyor uses magnetic north

Nighttimers used the services of another surveyor. His north and east directions were based on the North Star. He too measured distances eastward from the center of the town square in meters (x' in meters) and sacred distances north in miles (y' in miles). His records were complete and accurate. Every corner of a plot appeared in his book with its two coordinates, x' and y' .

Nighttime surveyor uses North Star north

One fall a student of surveying turned up with novel openmindedness. Contrary to all previous tradition he attended both of the rival schools operated by the two leaders of surveying. At the day school he learned from one expert his method of recording the location of the gates of the town and the corners of plots of land. At night school he learned the other method. As the days and nights passed the student puzzled more and more in an attempt to find some harmonious relationship between the rival ways of recording location. He carefully compared the records of the two surveyors on the locations of the town gates relative to the center of the town square:

Table 1. Two different sets of records for the same points.

<i>Place</i>	<i>Daytime surveyor's axes oriented to magnetic north (x in meters; y in miles)</i>		<i>Nighttime surveyor's axes oriented to the North Star (x' in meters; y' in miles)</i>	
Town square	0	0	0	0
Gate A	x_A	y_A	x'_A	y'_A
Gate B	x_B	y_B	x'_B	y'_B
Other gates	...	...	...	...

In defiance of tradition, the student took the daring and heretical step to convert northward measurements, previously expressed always in miles, into meters by multiplication with a constant conversion factor, k . He then discovered that the quantity $[(x_A)^2 + (ky_A)^2]^{1/2}$ based on Daytime measurements of the position of gate A had exactly the same numerical value as the quantity

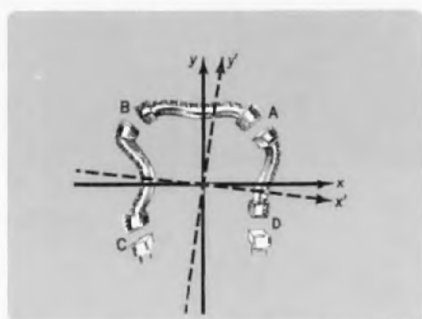


Fig. 1. The town and its gates, showing coordinate axes used by two different surveyors.

$[(x_A')^2 + (ky_A')^2]^{1/2}$ computed from the readings of the Nighttime surveyor for gate A. He tried the same comparison on the readings computed from the recorded positions of gate B, and found agreement here too. The student's excitement grew as he checked his scheme of comparison for all the other town gates and found everywhere agreement. He decided to give his discovery a name. He called the quantity

$$(1) \quad [(x)^2 + (ky)^2]^{1/2}$$

the *distance* of the point (x, y) from the center of town. He said that he had discovered the *principle of the invariance of distance*; that one gets exactly the same distances from the Daytime coordinates as from the Nighttime coordinates, despite the fact that the two sets of surveyors' numbers are quite different.

This story illustrates the naive state of physics before the discovery of *special relativity* by Einstein of Bern, Lorentz of Leiden, and Poincaré of Paris. How naive?

1. Surveyors in this mythical kingdom measured northward distances in a sacred unit, the mile, different from the unit used in measuring eastward distances. Similarly, people studying physics measured time in a sacred unit, the second, different from the unit used in measuring space. No one thought of using the same unit for both, or of what one could learn by squaring and combining space and time coordinates when both were measured in meters. The conversion factor between seconds and meters, namely the speed of light, $c = 2.997925 \times 10^8$ meters per second, was regarded as a sacred number. It was not recognized as a mere conversion factor like the factor of conversion between miles and meters—a factor that arose out of historical accidents alone, with no deeper physical significance.
2. In the parable the northbound coordinates, y and y' , as recorded by the two surveyors did not differ very much because the two directions of north were separated only by the small angle of 10 degrees. At first our mythical student thought the small differences between y and y' were due to surveying error alone. Analogously, people have thought of the time between the explosion of two firecrackers as the same, by whomever observed. Only in 1905 did we learn that the time difference between the second event and the first, or "reference event," really has dif-

ferent values. Think of an observer at the front end of the back door. (event") the reference event measurement later than the further down and its position events also the two observers

Table 2. S

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Reference event
Event A
Event B
Other events

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Discovery: invariance
of distance

ferent values, t and t' , for observers in different states of motion. Think of one observer standing quietly in the laboratory. The other observer zooms by in a high-speed rocket. The rocket comes in through the front entry, goes down the middle of the long corridor and out the back door. The first firecracker goes off in the corridor ("reference event") then the other ("event A"). Both observers agree that the reference event establishes the zero of time and the origin for distance measurements. The second explosion occurs, for example, 5 seconds later than the first, as measured by laboratory clocks, and 12 meters further down the corridor. Then its time coordinate is $t_A = 5$ seconds and its position coordinate is $x_A = 12$ meters. Other explosions and events also take place down the length of the corridor. The readings of the two observers can be arranged as in Table 2.

One observer uses
laboratory frame

Another observer uses
rocket frame

Table 2. Space and time coordinates of the same events as seen by two observers in relative motion. For simplicity the y and z coordinates are zero, and the rocket is moving in the x direction.

Event	Coordinates as measured by observer who is			
	standing (x in meters; t in seconds)		moving by in rocket (x' in meters; t' in seconds)	
Reference event	0	0	0	0
Event A	x_A	t_A	x'_A	t'_A
Event B	x_B	t_B	x'_B	t'_B
Other events	...	...	...	...

3. The mythical student's discovery of the concept of distance is matched by the Einstein-Poincaré discovery in 1905 of the idea of *interval*. The interval as calculated from the one observer's measurements

$$(2) \quad \text{interval} = [(ct_A)^2 - (x_A)^2]^{1/2}$$

agrees with the interval as calculated from the other observer's measurements

$$(3) \quad \text{interval} = [(ct'_A)^2 - (x'_A)^2]^{1/2}$$

Discovery: invariance
of interval

even though the *separate coordinates* employed in the two calculations *do not* agree. The two observers will find different space and time coordinates for events A, B, C, ... relative to the same reference event, but when they calculate the Einstein *intervals* between these events, their results will agree. *The invariance of the interval*—its independence from the choice of the reference frame—forces one to recognize that time cannot be separated from space. Space and time are part of the single entity, *spacetime*. The geometry of spacetime is truly four-dimensional. In one way of speaking, the "direction of the time axis" depends upon the state of motion of the observer, just as the directions of the y axes employed by the surveyors depend upon their different standards of "north."

The rest of this chapter is an elaboration of the analogy between surveying in space and relating events to one another in spacetime. Table 3 is a preview of this elaboration. To recognize the unity of space and time one follows the procedure that makes a landscape take on meaning—he looks at it from several angles. This is the reason for comparing space and time coordinates of an event in two *different* reference frames in relative motion.

Table 3. Preview: Elaboration of the parable of the surveyors.

<i>Parable of the surveyors: geometry of space</i>	<i>Analogy to physics: geometry of spacetime</i>
The task of the surveyor is to locate the position of a point (gate A) using one of two coordinate systems that are rotated relative to one another.	The task of the physicist is to locate the position and time of an event (firecracker explosion A) using one of two reference frames which are in motion relative to one another.
The two coordinate systems: oriented to magnetic north and to North-Star north.	The two reference frames: the laboratory frame and the rocket frame.
For convenience all surveyors agree to make position measurements with respect to a common origin (the center of the town square).	For convenience all physicists agree to make position and time measurements with respect to a common reference event (explosion of the reference firecracker).
The analysis of the surveyors' results is simplified if x and y coordinates of a point are both measured in the same units, in meters.	The analysis of the physicists' results is simplified if the x and t coordinates of an event are both measured in the same units, in meters.
The <i>separate</i> coordinates x_A and y_A of gate A <i>do not</i> have the same values respectively in two coordinate systems that are rotated relative to one another.	The <i>separate</i> coordinates x_A and t_A of event A <i>do not</i> have the same values respectively in two reference frames that are in uniform motion relative to one another.
<i>Invariance of distance.</i> The distance $(x_A^2 + y_A^2)^{1/2}$ between gate A and the town square has the same value when calculated using measurements made with respect to either of two rotated coordinate systems (x_A and y_A both measured in meters).	<i>Invariance of the interval.</i> The interval $(t_A^2 - x_A^2)^{1/2}$ between event A and the reference event has the same value when calculated using measurements made with respect to either of two reference frames in relative motion (x_A and t_A both measured in meters).
<i>Euclidean transformation.</i> Using Euclidean geometry, the surveyor can solve the following problem: <i>Given</i> the Nighttime coordinates x_A' and y_A' of gate A and the relative inclination of respective coordinate axes, <i>find</i> the Daytime coordinates x_A and y_A of the same gate.	<i>Lorentz transformation.</i> Using Lorentz geometry, the physicist can solve the following problem: <i>Given</i> the rocket coordinates x_A' and t_A' of event A and the relative velocity between rocket and laboratory frames, <i>find</i> the laboratory coordinates x_A and t_A of the same event.

The parable of the surveyors cautions us to use the same unit to measure both distance and time. So use meters for both. Time can be measured in meters. When a mirror is mounted at each end of a stick one-half meter long, a flash of light may be bounced back and forth between these two mir-

rors. Such a device light flash arrived traveled a round ticks of this clock of time. (Show light-travel time

One purpose To do this here respect to which gravity acts on motion as we know and other completely freely falling relational forces called an *inertial* laws of physics

The principle much simpler than mobile. Yet both with insufficient best minds of time not because nature grow established won. The concept it easy to think The problem of intuition—a probable number of to be perfectly

2. The Inert

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†Einstein, in a

‡For a comprehensive of relativity, together Reprints, published by the Institute of Physics, 33

§Paperback edition published in the Great

rors. Such a device is a *clock*. This clock may be said to "tick" each time the light flash arrives back at the first mirror. Between ticks the light flash has traveled a round-trip distance of 1 meter. Therefore the unit of time between ticks of this clock is called *1 meter of light-travel time* or more simply *1 meter of time*. (Show that 1 second is approximately equal to 3×10^8 meters of light-travel time.)

One purpose of the physicist is to sort out simple relations between events. To do this here he might as well choose a particular reference frame with respect to which the laws of physics have a simple form. Now, the force of gravity acts on everything near the earth. Its presence complicates the laws of motion as we know them from common experience. In order to eliminate this and other complications, we will, in the next section, focus attention on a freely falling reference frame near the earth. In this reference frame no gravitational forces will be felt. Such a gravitation-free reference frame will be called an *inertial reference frame*. Special relativity deals with the classical laws of physics expressed with respect to an inertial reference frame.

The principles of special relativity are remarkably simple. They are very much simpler than the axioms of Euclid or the principles of operating an automobile. Yet both Euclid and the automobile have been mastered—perhaps with insufficient surprise—by generations of ordinary people. Some of the best minds of the twentieth century struggled with the concepts of relativity, not because nature is obscure, but simply because man finds it difficult to outgrow established ways of looking at nature. For us the battle has already been won. The concepts of relativity can now be expressed simply enough to make it easy to think correctly—thus "making the bad difficult and the good easy."[†] The problem of understanding relativity is no longer one of *learning* but one of *intuition*—a practiced way of seeing. When seen with this intuition, a remarkable number of otherwise incomprehensible experimental results are revealed to be perfectly natural.[‡]

Simplify: Pick freely falling laboratory

2. The Inertial Reference Frame

Less than a month after the surrender at Appomattox ended the American Civil War (1861–65), the French author Jules Verne began writing *A Trip from the Earth to the Moon* and *A Trip around the Moon*.[§] Eminent American cannon designers, so the story goes, cast a great cannon in a pit dug in the earth of Florida with the cannon muzzle pointing skyward. From this cannon is fired a 10-ton projectile containing three men and several animals. As the projectile coasts outward in unpowered flight toward the moon after leaving the cannon, its passengers walk normally inside the projectile on the side

[†]Einstein, in a similar connection, in a letter to the architect Le Corbusier.

[‡]For a comprehensive set of references to introductory literature concerning the special theory of relativity, together with several reprints of articles, see *Special Relativity Theory*, Selected Reprints, published for the American Association of Physics Teachers by the American Institute of Physics, 335 East 45th Street, New York 17, New York, 1963.

[§]Paperback edition published by Dover Publications, New York. Hardcover edition published in the Great Illustrated Classics Series by Dodd, Mead and Company, New York, 1962.



Fig. 2. Illustration from an early edition of *A Trip around the Moon*. Satellite is the name of the unfortunate dog.

Passenger felt weight in Jules Verne's space ship

nearer the earth (Fig. 3, A). As the trip continues, the passengers find themselves pressed less and less against the floor of the space ship until finally, at the point where the earth and moon exert equal but opposite gravitational attraction for all objects, the passengers float free of the floor. Later, as the ship nears the moon, they walk around once again, but now against the side of the space ship nearer the moon. Early in the trip one of the dogs in the ship had died from injuries sustained at takeoff. The passengers had disposed of the remains of the dog through a scuttle in the side of the space ship, only to find that the corpse continues to float outside the window during the entire trip.

Paradox of passenger and dog

This story leads to a paradox of crucial importance to relativity. Verne thought it reasonable that the gravitational attraction of the earth would keep a passenger pressed against the earth side of the space ship during the early part of the trip. He also thought it reasonable that the dog should remain next to the ship, since both ship and dog independently follow the same path through space. But if the dog floats *outside* the space ship during the entire trip, why doesn't the passenger float around *inside* the space ship? If the ship were sawed in half would the passenger, now "outside," float free of the floor?

Passenger is weightless in real space ship

Our experience with actual space flights enables us to resolve this paradox. Jules Verne was in error about the motion of the passenger inside the space ship. Like the dog outside the ship, the passenger inside independently follows the same path through space as the space ship itself. Therefore he floats freely relative to the ship during the entire trip (Fig. 3, B). It is true that the gravitational field of the earth acts on the passenger. But it also acts on the space ship. In fact, with respect to the earth, the acceleration of the *spaceship* in the gravitational field of the earth is just *equal* to the acceleration of the

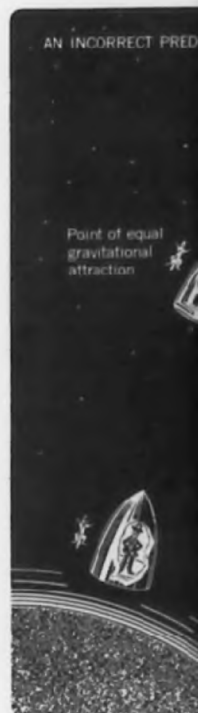


Fig. 3, A. Jules Verne in a free projectile with the projectile nearest whichever had greater attraction—but that the dog side the projectile during

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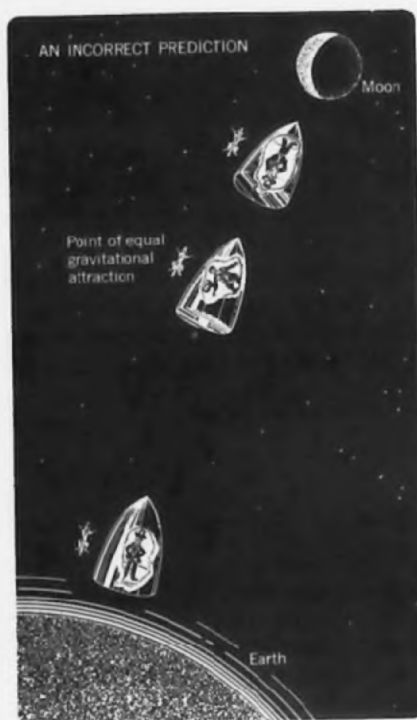


Fig. 3. A. Jules Verne believed that a passenger in a free projectile would stand on the side of the projectile nearest to the earth or moon, whichever had greatest gravitational attraction—but that the dog would float along beside the projectile during the entire trip.

Fig. 3. B. Correct prediction is that a passenger would float with respect to the projectile during entire trip. Verne was correct about the motion of the dog.

passenger in the gravitational field of the earth. Because of the equality of these accelerations there will be no *relative* acceleration between passenger and space ship. Thus the space ship serves as a reference frame ("inertial reference frame") relative to which the passenger does not experience an acceleration.

To say that the acceleration of the passenger relative to the space ship is zero is not to say that his velocity relative to it is necessarily also zero. He may have jumped from the floor or sprung from the side—in which case he will hurtle across the space and strike the opposite wall. However, when he has zero initial velocity relative to the ship the situation is particularly interesting, for he will also have zero velocity relative to it at all later times. He and the ship will follow identical paths through space. How remarkable that the passenger who cannot see the outside nevertheless moves on this deterministic orbit. Without a way to control his motion and even with his eyes closed he will not touch the wall. How could one do better at eliminating gravitational influences!

A modern space ship carrying a passenger is shot vertically from the earth, rises, and falls back toward the earth (Fig. 4). (The passenger of an elevator car experiences a close approximation to this fall when the elevator cable is cut!) Choose this freely falling space ship as the best possible reference frame in which to do physics. This reference frame is best because, among other

Concept of inertial reference frame



Fig. 4. Space ship in free fall near the earth.



Fig. 5. Railway coach in free fall in horizontal position near the earth.



Fig. 6. Railway coach in free fall in vertical position near the earth.

things, the laws of motion of a particle are simple in a falling space vehicle. A free particle at rest in the vehicle remains at rest in the vehicle. When the particle is given a gentle push, it moves across the vehicle in a straight line with constant speed. Further experiments show that *all* the laws of mechanics can be expressed simply with respect to a falling space ship. We call such a space ship that rises or falls freely—or more generally moves freely in space—an *inertial reference frame*.

Look at the freely falling space ship from the surface of the earth. There is a simple reason why the free particle at rest relative to the space ship remains at rest in the space ship. This reason is that, with respect to the surface of the earth, the particle and the space ship both fall with the same acceleration (Fig. 4). It is because of this equal acceleration that the *relative* positions of the particle and the space ship do not change if the particle is originally at rest in the space ship.

The definition of an inertial frame requires that *no gravitational forces will be felt in it*. If such a reference frame is to be a space ship near the earth, it cannot be a very large one because widely separated particles within it will be differently affected by the *nonuniform* gravitational field of the earth. For example, particles released side by side will each be attracted toward the center of the earth, so they will move closer together as observed from the falling space ship (Fig. 5). As another example, think of the two particles being released far apart vertically but directly above one another (Fig. 6). Their gravitational accelerations toward the earth will be in the same direction. However the particle nearer the earth will slowly leave the other one behind: the two particles will move farther apart as the space ship falls. In either of these instances the laws of mechanics will not be simple in a very large space ship: the large space ship will not be an inertial frame.

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nonuniform: large
space ship is not
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in a very large space

Now, we want the laws of mechanics to look simple in the space ship. Therefore we want to eliminate all relative accelerations produced by external causes—"eliminate" meaning to reduce these accelerations below the limit of detection so that they will not interfere with the more important accelerations we wish to study, such as those produced when two particles collide. This can be done by choosing a space ship that is sufficiently small. The smaller the space ship, the smaller will be the relative accelerations of objects at different points in the space ship. Let someone have instruments for the detection of relative accelerations with any given degree of sensitivity. No matter how fine that sensitivity, the space ship can always be made so small that these perturbing relative accelerations are too small to be detectable. Within these limits of sensitivity the space ship is then an *inertial reference frame*.

When is a space ship or any other vehicle small enough to be called an inertial reference frame? Or when is the relative acceleration of free particles at opposite ends of the vehicle too slight to be detected? Analyzing the conditions inside one vehicle will serve to illustrate these considerations. A railway coach 25 meters long is dropped in a *horizontal* position from a height of 250 meters onto the surface of the earth (Fig. 5). The time from release to impact is about 7 seconds, or about 21×10^8 meters of light-travel time. Let tiny ball bearings be released initially from rest—and in mid-air—at opposite ends of the coach. Then, during the time of fall, they will move *toward* each other a distance of 10^{-3} meters—the thickness of 9 pages of this book—because of the difference in *direction* of the earth's gravitational pull upon them (see Ex. 32). As another example, assume that the same railway coach is dropped in a *vertical* position, and that the lower end of the coach is initially 250 meters from the surface of the earth (Fig. 6). Again two tiny ball bearings are released from rest at opposite ends of the coach. In this case, during the time of fall, the ball bearings will move *apart* by a distance of 2×10^{-3} meters because of the greater gravitational acceleration of the one nearer the earth. In either of these examples let the measuring equipment in use in the coach be just short of the sensitivity required to detect the relative motion of the ball bearings. Then, with equipment of this degree of sensitivity, and with the limited time of observation, the railway coach—or, to use an earlier example, the freely falling space ship—serves as an inertial reference frame. When the sensitivity of the measuring equipment is increased, then the space ship will not serve as an inertial reference frame unless changes are made. Either the 25-meter domain in which observations are made must be shortened, or the time given to the observations must be decreased. Or, better, some appropriate combination of the space and time dimensions of the region under observation must be cut down. Or, as a final alternative, the whole apparatus must be shot by a rocket (part c of Ex. 32) up to a region of space where one cannot detect the "differential in the gravitational acceleration" between one side of the coach and another—to use one way of speaking. In another way of speaking, the accelerations of the particles *relative to the coach* must be too small to be perceived. These relative accelerations can be measured from inside the coach without observing anything external. Only when these relative accelerations are too small to be detected is there a reference frame with respect to which the laws of motion are simple—an *inertial reference frame*.

A reference frame is said to be inertial in a certain region of space and time when, throughout that region of spacetime, and within some specified accuracy, every test particle that is initially at rest remains at rest, and every test particle that is initially in motion continues that motion without change in speed or in

*Example of space ship
small enough to be
inertial frame*

*Inertial reference
frame defined*

direction. An inertial reference frame is also called a *Lorentz reference frame*. In terms of this definition, inertial frames are necessarily always *local* ones, that is, inertial in a limited region of spacetime.

"Region of spacetime." What is the precise meaning of this term? The long narrow railway coach in the example served as a means to probe spacetime for a limited stretch of time and in one or another single direction in space. It can be oriented north-south, or east-west, or up-down. Whatever the orientation, the relative acceleration of the tiny ball bearings released at the two ends can be measured. For all three directions—and for all intermediate directions—let it be found by calculation that the relative drift of the two test particles is half the minimum detectable amount or less. Then throughout a cube of space 25 meters on an edge and for a lapse of time of 7 seconds, test particles moving every which way depart from straight-line motion by undetectable amounts.

Region of spacetime
defined

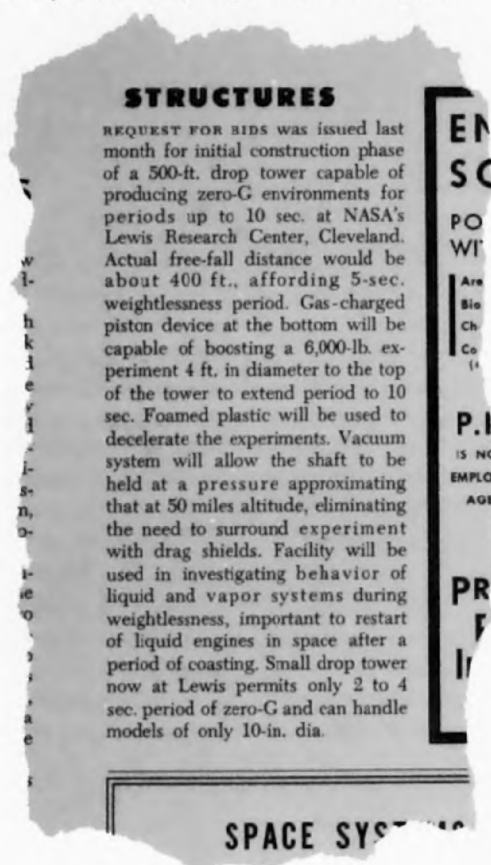


Fig. 7. Modern inertial reference frame. From *Engineering Opportunities*, March 1964.

In other words, the reference frame is inertial in a region of spacetime with dimensions

$$(25 \text{ meters} \times 25 \text{ meters} \times 25 \text{ meters of space}) \times (21 \times 10^8 \text{ meters of time})$$

For a discussion of spacetime regions larger than those of local inertial frames, see Chapter 3.

"Test particle."

It must have so little mass that it will not affect the mechanics the gravitation must be negligible. A particle of mass 10⁻¹⁰ tenth meter from the center must undergo a displacement in this sense—a test particle—it does not itself produce a field.

It would be impossible to observe any feature of nature that all fall with the same acceleration inside a falling space. Even when they are not remain at rest; according to the definition of different substances. The legend Galileo dropped a stone from Pisa in order to verify an accuracy of five parts to platinum. More reliable than the earth (see Ex. 35). The fantastic amplifications due to the three parts in a hundred. Peter G. Roll.† The principle in all of physics is the kind of test particle.

It follows from the definition of a test particle to determine that is inertial for all purposes.

3. The Principle

We describe the motion of a particle in a reference frame in order to determine the motion of particles and—if possible—to refer to more than one particle. It is carried by a spacecraft of second order of approximation.

†On the question of the definition of a test particle, see *Science* by Lloyd W.

‡See the chapter on the theory of relativity, edited by C. and B. by Dicke *The Theory of Relativity* (New York, 1964).

orentz reference frame.
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undetectable amounts.

“Test particle.” How small must a particle be to qualify as a test particle? It must have so little mass that, within some specified accuracy, its presence will not affect the motion of other nearby particles. In terms of Newtonian mechanics the gravitational attraction of the test particle for other particles must be negligible within the accuracy specified. As an example, consider a particle of mass 10 kilograms. A second and less massive particle placed one-tenth meter from it and initially at rest will, in less than three minutes, undergo a displacement of 10^{-3} meters. Thus the 10-kilogram object is not—in this sense—a test particle. A test particle *responds* to gravitational forces but it does not itself *produce* any significant gravitational force.

*Test particle
defined*

It would be impossible to define an inertial reference frame if it were not for a remarkable feature of nature. Particles of different size, shape, and material in the same location all fall with the *same acceleration* toward the earth. If this were not so, an observer inside a falling space ship would notice a relative acceleration among different particles even when they are close together; at least some of the particles initially at rest would not remain at rest; that is, the space ship would not be an inertial reference frame according to the definition. How sure are we that particles in the *same* location but of *different* substances all fall toward the earth with the same acceleration? According to legend Galileo dropped balls made of different materials from the Leaning Tower of Pisa in order to verify this assumption.† In 1922 Baron Roland von Eötvös checked to an accuracy of five parts in 10^9 that the earth imparts the same acceleration to wood and to platinum. More recently Robert H. Dicke has pointed out that the sun is more suitable than the earth as source for the gravitational acceleration that one will measure (see Ex. 35). The alternation in direction of the sun's pull every 12 hours lends itself to fantastic amplification by resonance. Cylinders of aluminum and gold experience accelerations due to the sun (0.59×10^{-2} meters per second per second) that are the same to three parts in a hundred thousand million (3 in 10^{11}), according to R. H. Dicke and Peter G. Roll.‡ This is one of the most sensitive checks of a fundamental physical principle in all of physics: the identity of the acceleration produced by gravity in every kind of test particle.

*Inertial frame is
definable because all
substances fall with
same acceleration*

It follows from this principle that a particle made of *any* material can be used as a test particle to determine whether a given reference frame is inertial. A reference frame that is inertial for one kind of test particle will be inertial for all kinds of test particles.

3. The Principle of Relativity

We describe the motion of test particles with reference to a particular reference frame in order to determine whether that frame is inertial. The same test particles and—if they collide—the same collisions may be described with reference to more than one inertial frame. The one reference frame might be carried by a space ship built like a hollow cylinder (Fig. 8,A), the other by a second craft of similar construction just enough smaller to zoom through in-

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Engineering Opportuni-
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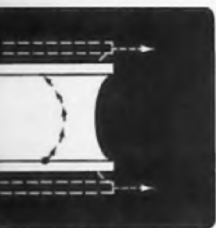
gion of spacetime with

$\times 10^8$ meters of time)

of local inertial frames,

†On the question whether Galileo actually performed this experiment, see *Physics the Pioneer Science* by Lloyd W. Taylor, (Dover Publications, New York, 1959), Vol. 1, p. 25.

‡See the chapter on experimental relativity by Dicke in *Relativity, Groups, and Topology*, edited by C. and B. DeWitt, (Gordon and Breach, New York, 1964), pp. 173–177 or the book by Dicke *The Theoretical Significance of Experimental Relativity* (Gordon and Breach, New York, 1964).



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reference frame

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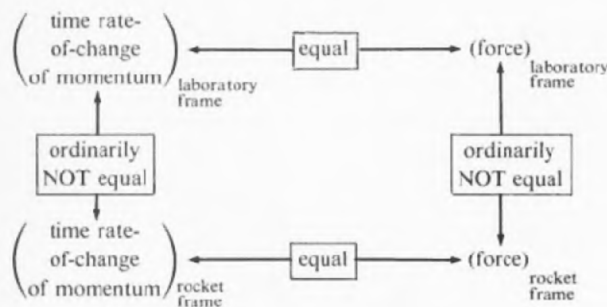
ial reference frame.

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ved in one inertial refer-
on in any other inertial

reference frame. Both the *form* of the laws of physics and the *numerical values* of the *physical constants* that these laws contain are the same in every inertial reference frame. All inertial frames are equivalent in terms of *every* law of physics. Expressed in negative terms, the principle of relativity says that *the laws of physics cannot provide a way to distinguish one inertial frame from another*—any more than the surveyor's tape and level give a means to tell North Star north from magnetic north!

Notice what the principle of relativity does *not* say. It does not say that the time between events A and B will appear the same when measured from two different inertial reference frames. Neither does it say that spatial separation between the two events will be the same in the two frames. Ordinarily neither times nor distances will be the same in the two frames—any more than the northward and eastward components of the separation of gates A and B as read by the Daytime surveyor are identical to those recorded by the Nighttime surveyor. In consequence the momentum of a given particle will have a value in one frame that is different from its value in a second frame. Even the time rate-of-change of momentum will ordinarily differ between the two frames. And so will the force. Thus, when studying the motion of a charged particle, two observers in relative motion will not necessarily find the same values for the electric field or the magnetic field acting upon this charged particle. The total force, produced by the electric and magnetic fields together, will differ between the one inertial reference frame and the other.

The physics that is so different between one frame and the other is nevertheless the same in the two frames! Physical quantities differ in *value* between the two frames but fulfill identical *laws*. The time rate-of-change of momentum in one frame is equal to the total force as measured in that frame (Newton's second law). The time rate-of-change of momentum in the second frame is equal to the total force as measured in that second frame:



Not only the laws of mechanics but also the laws of electromagnetism and all other laws of physics hold true as well in one inertial reference frame as in any other inertial reference frame. This is what it means to say that "the laws of physics cannot provide a way to distinguish one inertial reference frame from another."

The laws of electromagnetism hold true as well in one inertial reference frame as in any other inertial reference frame. The numerical value of the speed of light, $c = 2.997925 \times 10^8$ meters per second, is one of the constants which

*What principle of rela-
tivity does NOT say!*

appears in the laws of electromagnetism. According to the principle of relativity this experimental value must be the same in each of two inertial reference frames in uniform relative motion. Has experiment shown this to be true? The answer is yes, but the experiments to date are much less sensitive than they should be for so important a question. For the moment let us pull in our horns and concentrate on a simpler question which *can* be answered definitively. The laws of electromagnetism contain no expressions that depend upon direction. Therefore one expects to find the round-trip speed of a flash of light to be the same whether the flash travels north-south or east-west: the speed of light is *isotropic*. But now let these same flashes of light be observed from a uniformly moving rocket. With respect to this rocket, will the round-trip speed of light not be different for light moving out and back along different lines? The principle of relativity says *no*: the speed of light, which is isotropic in one inertial frame, is also isotropic in all other inertial frames that share the same spacetime region.

How strange this result is! We know that the speed of sound in air is the same in all directions if the air is still. But let a stiff wind be blowing—or, to get the same result, move through the still air in an automobile. Then, with respect to the automobile, the “downstream speed of sound” is greater than the “upstream speed of sound.” And a simple calculation shows that both these speeds are different from the speed of sound measured across the wind. The round-trip speed of sound measured with respect to the automobile will be different in different directions. The same result is true for every other form of wave motion we know about—except that of light! How can we be so sure that this result is *not* true for experiments with light? Our assurance is based on a series of refined experiments beginning with the classic experiment of A. A. Michelson and E. W. Morley performed after 1880.† They used the earth itself as a moving reference frame. (The earth is effectively an inertial frame for local experiments with light—see Ex. 31.) The earth moves at a speed of about 30 kilometers per second in its orbit about the sun. In essence, Michelson and Morley compared the round-trip speed of light along the line of the earth’s motion with the speed of light perpendicular to this line. They repeated this experiment at different times of the year, when the earth was moving in different directions with respect to the fixed stars. No effect of the motion of the earth on the relative speed of light in the two perpendicular directions was observed. From the accuracy of their experiment they determined that the measured speed of light in the two perpendicular directions was the same to a sensitivity of one-sixth of the orbital speed of the earth (see Ex. 33). More recent experiments have reduced this uncertainty to three percent of the orbital speed of the earth.‡ The Michelson-Morley experiment and its modern improvements tell us that in every inertial frame the round-trip speed of light is the same in every direction—the speed of light is *isotropic* in both laboratory and rocket frames as predicted by the principle of relativity. But the principle of relativity says more than this. Not only must the speed of light be isotropic in the laboratory frame—and also isotropic in the rocket frame—but also, if

*Michelson-Morley
experiment: speed of
light is isotropic in all
inertial frames*

†A. A. Michelson and E. W. Morley, *American Journal of Science*, **34**, 333 (1887).

‡T. S. Jaseja, A. Javan, J. Murray, and C. H. Townes, *Physical Review*, **133**, A1221 (1964).

Table 4. Modern
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Table 4. Modern tests to answer the question, "Does the round-trip speed of light differ between one reference frame and another?"

TWO REFERENCE FRAMES	
ONE REFERENCE FRAME	The earth moving in one direction around the sun in, say, January.
ANOTHER REFERENCE FRAME	The earth moving in the opposite direction (with respect to the fixed stars) in July.
EXPERIMENTAL RESULTS	
RESULT OF THE MICHELSON-MORLEY EXPERIMENTS	
<i>Original experiment</i>	
Observers in neither frame (may be the <i>same</i> observer on earth who repeats the experiment after waiting six months) can detect differences in the round-trip speed of light in any two perpendicular directions greater than one-sixth of the speed of the earth in its orbit.	
<i>The more modern experiment</i>	
Observers in neither frame can detect differences in the round-trip speed of light in any two perpendicular directions greater than three percent of the speed of the earth in its orbit.	
RESULT OF THE KENNEDY-THORNDIKE EXPERIMENT	
The round-trip speed of light is the same in one of the seasonal reference frames defined above as in the other reference frame with a sensitivity of about two meters per second.	
INTERPRETATION OF THE EXPERIMENTS	
THE MORE MODERN MICHELSON-MORLEY EXPERIMENT	
The speed of the earth in its orbit about the sun is 30 kilometers per second = $1/10,000$ of the speed of light	
Thus the <i>difference</i> of the round-trip speed of light measured in two perpendicular directions is less than $3/100$ of $1/10,000$ of the speed of light which is less than $3/1,000,000$ of the speed of light	
Therefore the <i>principle of relativity</i> is supported by this modern experiment with a sensitivity of three parts in a million	
THE KENNEDY-THORNDIKE EXPERIMENT	
The <i>difference</i> of the round-trip speed of light as measured in the two frames is less than about 2 meters per second which is less than $1/100,000,000$ of the speed of light	
Therefore the <i>principle of relativity</i> is supported by this experiment with a sensitivity of one part in a hundred million	

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*Kennedy-Thorndike
experiment: speed of
light has same
numerical value in all
inertial frames*

this principle is correct, the *numerical value* of this isotropic speed, $c = 2.997925 \times 10^8$ meters per second, must be the same in the rocket frame as in the laboratory frame. Can this prediction also be verified by experiment? This verification was carried out by R. J. Kennedy and E. M. Thorndike about 50 years after Michelson and Morley did their experiment.[†] Like Michelson and Morley, Kennedy and Thorndike used the earth as a moving reference frame. They tried to detect any variation in the magnitude of the round-trip speed of light as the earth moved in different directions around the sun at different times of the year. From the accuracy of their negative results one can conclude that there is no difference as great as about two meters per second in the round-trip speed of light as between two reference frames with a relative velocity of 60 kilometers per second (twice the speed of the earth in its orbit; see Ex. 34). In the Kennedy-Thorndike experiment the standard of length is the interferometer base itself, a single block of fused quartz kept in a vacuum at a temperature constant to about a thousandth of a degree. The standard of time is provided by the characteristic vibration period associated with a particular green spectral line of a mercury atom. Keeping conditions constant for months constituted the most important single difficulty—and difference—of this Pasadena, California experiment as contrasted to the Cleveland, Ohio Michelson-Morley experiment, where the relevant comparisons (one direction as against another) could be made in the course of a single day. Table 4 summarizes the conclusions of the Michelson-Morley and the Kennedy-Thorndike experiments.

Although the sensitivity of neither of these experiments is as great as that of the Eötvös-Dicke experiment (three parts in a hundred thousand million), the results are nonetheless striking experimental support for the principle of relativity. Happily, there are plans to improve the sensitivity of the Kennedy-Thorndike experiment.[‡] This improvement in sensitivity is important. The measurement of time in meters of light-travel time has meaning only if light travels one meter in the same time in all frames. The equality of the speed of light in rocket and laboratory frames provides a simple way (Section 5) to compare clocks between the two frames. This comparison depends for its validity on the null result of the Kennedy-Thorndike experiment.

In 1905 the principle of relativity was a shocking heresy, which offended the intuition and common-sense way of looking at nature of most physicists. It has taken a long time to become accustomed to the apparently absurd idea that one particular speed has the same value measured in two overlapping inertial frames in relative motion. The principle of relativity is used every day in many fields of physics where it is continually under severe tests. For example, the Stanford Linear Electron Accelerator (estimated cost: \$300,000,000) has to be two miles long to push electrons up to a speed that is almost the speed of light (the difference from the speed of light is only 8 parts in 10^{11}). If the pre-Einstein Newtonian laws of mechanics were correct, then the accelerator

*Structure of spacetime
makes LINAC cost
\$300,000,000*

[†]R. J. Kennedy and E. M. Thorndike, *Physical Review*, **42**, 400 (1932).

[‡]See Jaseja, Javan, Murray, and Townes, *Physical Review*, **133**, A1221 (1964). For a careful analysis of the experimental foundations of special relativity see H. P. Robertson, "Postulate versus Observation in the Special Theory of Relativity," *Reviews of Modern Physics*, **21**, 378 (1949).

would need to be less than the same speed!

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would need to be less than one inch long (Ex. 55) to produce electrons with the same speed!

4. The Coordinates of an Event

The inertial reference frame is to a student of physics what the north-south east-west grid of lines in a township is to a surveyor. The surveyor is concerned with position in space. The student of physics is concerned with location of an event in space and in *time*. The Daytime and Nighttime surveyors *could* have dispensed with north-south and east-west coordinates and simply measured the *distance* between any two gates; but at the start they did not even know there was any such quantity as "distance." In the same way in this chapter we could have gone about locating events in spacetime solely by measuring the *intervals* between one event and another, without any regard for "space" and "time" coordinates individually.† However, we have to start as physics did before 1905, without benefit of any concept of interval. This concept will force itself upon our attention as the concept of distance forced itself upon the surveyors. The two men measured north-south and east-west coordinates in two different coordinate systems. Only later did they see the connection ("invariance of the distance") between the very different numbers in their notebooks. Similarly, we will begin with space and time coordinates of events in the laboratory reference frame, and space and time coordinates of the same events in the rocket reference frame. Then there will be a firm basis for concluding that the interval between two events as determined from laboratory numbers is identical with the interval between the same two events as calculated from the very different rocket readings ("invariance of the interval").

The fundamental concept in surveying is a *place*. The fundamental concept in physics is an *event*. An event is specified not only by a place but also by a time of happening. Some examples of events are: emission of particles or flashes of light (explosions), reflection or absorption of particles or light flashes, collisions, and near-collisions called *coincidences*.

How can one determine the place and time at which an event occurs in a given inertial reference frame? Think of constructing a frame by assembling meter sticks into a cubical latticework similar to the "jungle gym" seen on playgrounds (Fig. 9). At every intersection of this latticework fix a clock. These clocks can be constructed in any way, but are *calibrated* in meters of light-travel time. In Section 1 we discussed how to obtain such a calibration by bouncing a flash of light back and forth between two mirrors one-half meter apart. This mirror clock is said to "tick" each time the light flash arrives back at the first mirror. Between ticks the light flash travels a round-trip distance of 1 meter: Call the unit of time between ticks *1 meter of light-travel time*, or more simply, *1 meter of time*. The speed of light in conventional units has the measured value $c = 2.997925 \times 10^8$ meters per second. Light will travel 1

Why use coordinates?

Event defined

Latticework of clocks

†Such a treatment is presented by Robert F. Marzke and John A. Wheeler in *Gravitation and Relativity*, edited by H.-Y. Chiu and W. F. Hoffmann, (W. A. Benjamin, New York, 1964).

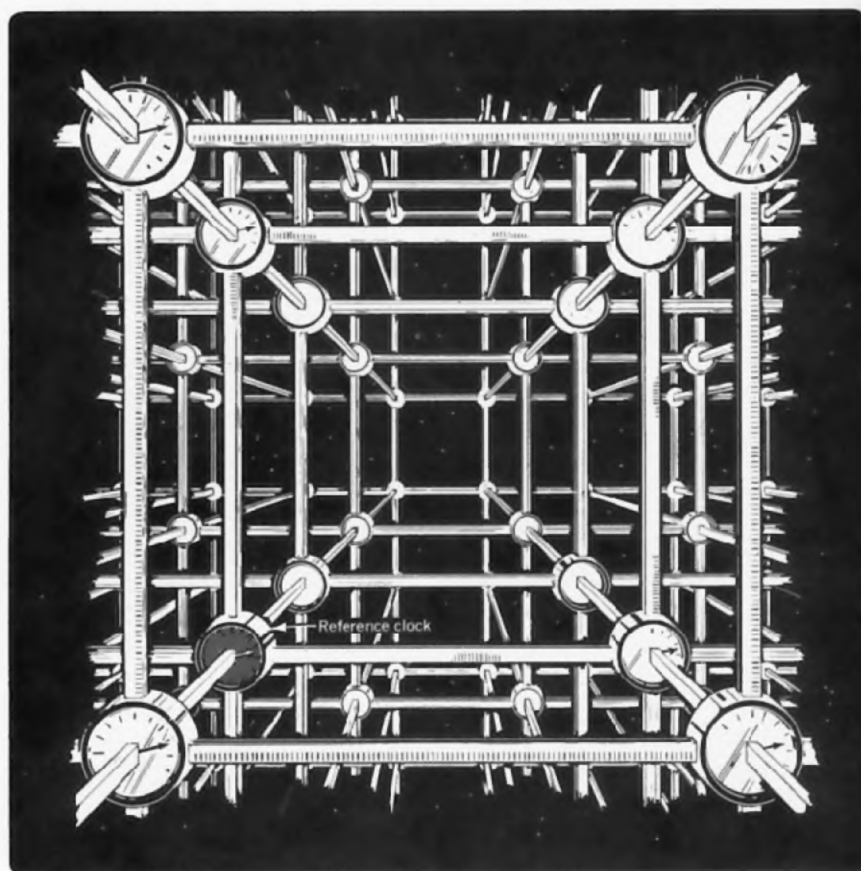


Fig. 9. Lattice of meter sticks and clocks.

meter in the time, $1 \text{ meter}/c = 3.335640 \times 10^{-9}$ seconds. Hence *1 meter of light-travel time is equal to 3.335640×10^{-9} seconds*—about 3.3 nanoseconds in the terminology of high-speed electronic circuits! We assume that *every* clock in the latticework, whatever its construction, has been calibrated in meters of light-travel time.

How are the different clocks in the lattice to be *synchronized* with one another? As follows: Pick one of the clocks in the lattice as the standard of time and take it to be the *origin* of an x, y, z coordinate system. Start this reference clock with its pointer at $t = 0$. At this instant let it send out a flash of light that spreads in all directions. Call this flash of light the *reference flash*. When the reference flash gets to a clock 5 meters away, we want that clock to read 5 meters of light-travel time. So an assistant *sets* that clock to 5 meters of time long before the experiment begins, *holds* it at 5 meters, and *releases* it only when the reference flash arrives. When assistants at all the clocks in the lattice have followed this procedure (each setting his clock to a time in meters equal to his own distance from the reference clock and starting it when the light flash arrives), the clocks in the lattice are said to be *synchronized*.

*Synchronizing clocks
in lattice*

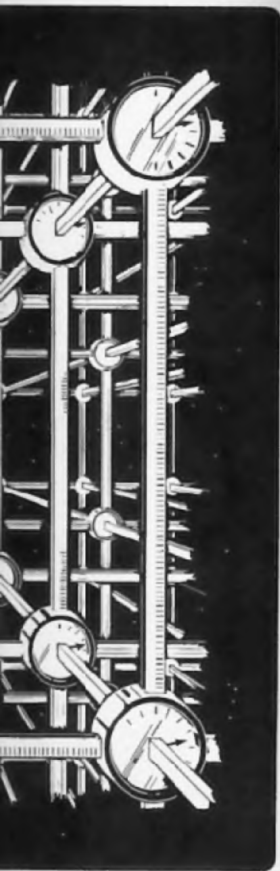
There are other portable clock carried around the procedure involves at a different rate remain at rest in the one when it is bro we use a moving cl speed of light, it is nization will give method. Moreover portable clock abo

The latticework tion and time at w taken to be the po will be taken to be coordinates of an e specify the *spatial* (meters) when the e are installed by a f be able to detect th Each will punch o the location of the and later analyzed

Why a lattice b lattice punches ou is 0.4 meters to th The location of t of a meter. The ti ciable fraction of a adequate for obser measurements on the lattice spacing 1 meter is a lattice high-energy accele appropriate. The l accuracy is desired

In relativity we At one place or al *speaking about the* inertial frame of re of the "ideal obser observer as the ma by all the recording we will hereafter b

The clocks reve that the particle p coordinate of this



conds. Hence 1 meter of
—about 3.3 nanoseconds in
e assume that every clock
en calibrated in meters of

synchronized with one an-
ce as the standard of time
system. Start this reference
nd out a flash of light that
reference flash. When the
want that clock to read 5
clock to 5 meters of time
eters, and releases it only
all the clocks in the lattice
o a time in meters equal to
ing it when the light flash
onized.

There are other possible ways to synchronize clocks. For example, an extra portable clock could be set to the reference clock at the origin and then carried around the lattice in order to set the rest of the clocks. However, this procedure involves a *moving* clock. We will see later that a moving clock runs at a different rate as measured by clocks in the lattice than do clocks that remain at rest in the lattice. The portable clock will not even agree with the first one when it is brought back next to it! (Clock paradox, Ex. 27). However, if we use a moving clock that travels at a speed that is a *very small fraction* of the speed of light, it is only slightly in error, and this second method of synchronization will give very nearly the same result as the first—and standard—method. Moreover the error can be made as small as desired by carrying the portable clock about sufficiently slowly.

The latticework of synchronized clocks can be used to determine the location and time at which any given event occurs. The *location* of the event will be taken to be the position of the clock nearest the event. The *time* of the event will be taken to be the time recorded on the lattice clock nearest the event. The *coordinates* of an event will then consist of four numbers: three numbers that specify the *spatial position* of the clock nearest the event and the *time* (in meters) when the event occurred as recorded by that clock. The clocks, if they are installed by a foresighted experimenter, will be *recording* clocks. Each will be able to detect the occurrence of an event (passage of a light flash or particle). Each will punch onto a card the nature of the event, the time of the event and the location of the clock. The cards can then be collected from all the clocks and later analyzed, perhaps by automatic equipment.

Why a lattice built of rods that are *one* meter long? When a clock in this lattice punches out a card, one will not know whether the event so recorded is 0.4 meters to the left of the clock, for instance, or 0.2 meters to the right. The location of the event will be uncertain by some substantial fraction of a meter. The time of the event will also be unknown within some appreciable fraction of a meter of light-travel time. This accuracy, however, is quite adequate for observing the passage of a rocket. It is extravagantly good for measurements on planetary orbits—it would even be reasonable to increase the lattice spacing from 1 meter to hundreds of meters. Neither 100 meters nor 1 meter is a lattice spacing suitable for studying the tracks of particles from a high-energy accelerator. There a centimeter or a millimeter would be more appropriate. The location and time of an event can be determined to whatever accuracy is desired by constructing a lattice with sufficiently small spacing.

In relativity we often speak about “the observer.” Where is this observer? At one place or all over the place? The word “observer” is a shorthand way of speaking about the whole collection of recording clocks associated with one inertial frame of reference. No one real observer could easily do what we ask of the “ideal observer” in our analysis of relativity. So it is best to think of the observer as the man who goes around picking up the punched cards turned out by all the recording clocks in his employ. This is the sophisticated sense in which we will hereafter be using the phrase “the observer finds such and such.”

The clocks reveal the motion of a particle through the lattice: each clock that the particle passes punches out the time of passage as well as the space coordinate of this event. How can the path of the particle be described in

*Latticework used to
measure the four
coordinates of event*

*Lattice spacing
depends on scale of
physics under study*

Observer defined

*Clock records reveal
motion of particle
through lattice*

terms of numbers? By recording the coordinates of these events along this path. Differences between the coordinates of successive events reveal the velocity of the particle. The conventional units for speed, v , are meters per second. However, when time is measured in meters of light-travel time, then speed is expressed in meters of distance covered per meter of time. To avoid confusion, speed expressed in meters per meter will be represented by the Greek letter beta, β . A flash of light moves one meter of distance in one meter of light-travel time, or, $\beta_{\text{light}} = 1$. The speed of other particles in meters per meter represents a fraction of the speed of light. In other words $\beta = v/c$. Here and hereafter c represents the speed of light.

Verifying that lattice
furnishes inertial frame

From the motion of test particles through the latticework of clocks—or rather from the records of coincidences punched out by the recording clocks—we can determine whether the latticework constitutes an inertial reference frame. If the records show (a) that—within some specified accuracy—a test particle moves consecutively past clocks that lie in a straight line, (b) that the speed β of the test particle calculated from the same records is constant—again, within some specified accuracy—and, (c) that the same results are true for as many test-particle paths as the most industrious observer cares to trace throughout the given region of space and time, then the lattice constitutes an inertial reference frame throughout that region of spacetime.

Laboratory and rocket
frames; x axes coincide

Once again we have described the motion of test particles with respect to a particular reference frame in order to determine whether that frame is inertial. The same test particles and—if they collide—the same collisions may be described with reference to one inertial frame as well as another. Let two reference frames be two different latticeworks of meter sticks and clocks, one moving uniformly relative to the other, and in such a way that their x axes coincide. Call one of these frames the *laboratory frame* and the other—moving in the positive x direction relative to the laboratory frame—the *rocket frame* (Figs. 10 and 11). The rocket is *unpowered* and coasts along with constant velocity relative to the laboratory. Let the rocket and laboratory latticeworks be *overlapping* in the sense that there is a region of spacetime common to both frames (as described in Section 3 and Fig. 8). Test particles move through this common region of spacetime. From the motion of these test particles as recorded by his own clocks, an observer in each frame verifies that his frame is *inertial*.

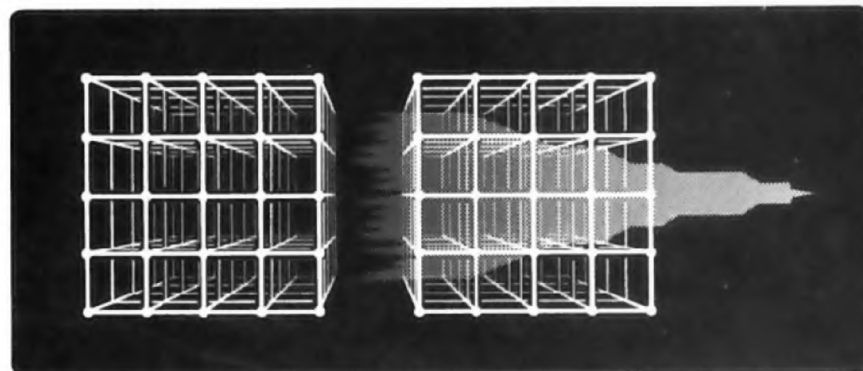


Fig. 10. Laboratory and rocket frames. The two latticeworks intermeshed a second ago.

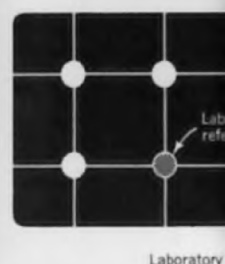


Fig. 11. Laboratory and clock of each frame is

A firecracker explosion in the laboratory lattice nearest the rocket lattice nearest the laboratory clock. One result can be seen in the recording laboratory clocks. To show this, let the marks on the laboratory clocks. In the special case, $y = 1$ meter, the laboratory y clock appears on the $y = 1$ meter. For suppose that the laboratory clocks: passed "inside" the rocket rods above the $y = 1$ meter. $y = 1$ rocket clock case there would be frames. But no one any other experimenter any such experimenter possible. Therefore we frames using this event such as the explosion in the rocket frame.

By a similar argument, an event in the laboratory frame as in the laboratory coordinate of an event

Fig. 12. Demonstration that an event is the same in both frames.

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ve events reveal the veloc-
 v , are meters per second.
travel time, then speed is
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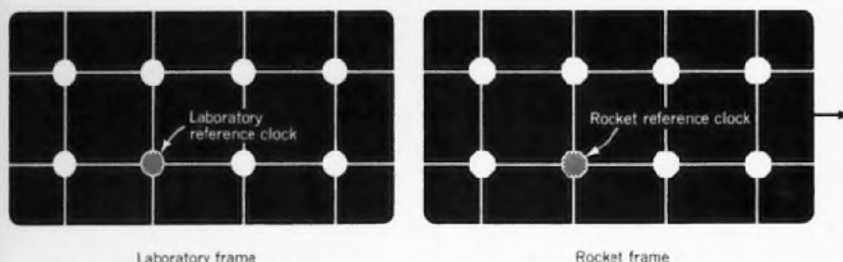


Fig. 11. Laboratory and rocket frames, further schematized from Fig. 10. The central reference clock of each frame is shaded.

A firecracker explodes. The explosion is recorded by the clock in the laboratory lattice nearest to the explosion. It is recorded also by the clock in the rocket lattice nearest to the explosion. How do the coordinates of the recording laboratory clock compare with the coordinates of the recording rocket clock? One result can be derived immediately from the principle of relativity: the recording laboratory and rocket clocks will have the same y coordinates. To show this, let the recording rocket clock carry a wet paint brush that makes marks on the laboratory lattice as it moves past. Figure 12 shows this for the special case, $y = 1$ meter. The marks on the laboratory lattice serve to measure the laboratory y coordinate of the $y = 1$ rocket clock. These paint marks appear on the $y = 1$ laboratory clocks rather than above them or below them. For suppose that the paint marks appear on the lattice rods below the $y = 1$ laboratory clocks: Then both observers will agree that the $y = 1$ rocket clocks passed "inside" the $y = 1$ laboratory clocks. Permanent paint marks would verify this for all to see. Similarly, if the paint marks appear on the lattice rods above the $y = 1$ laboratory clocks, both observers will agree that the $y = 1$ rocket clocks passed "outside" the $y = 1$ laboratory clocks. In either case there would be a way to distinguish experimentally between the two frames. But no one has been able to distinguish between these two frames using any other experiment. The principle of relativity embodies the assumption that any such experimental distinction between inertial reference frames is impossible. Therefore we assume that no one could distinguish between the two frames using this experiment. It follows that the y coordinate of any event—such as the explosion that began this paragraph—will have the same y coordinate in the rocket frame as in the laboratory frame.

By a similar argument the z coordinate of an event is the same in the rocket frame as in the laboratory frame. Notice that both the y coordinate and the z coordinate of an event are measured in a direction perpendicular to the direc-

Laboratory and rocket
observers record single
event

y coordinate of event is
same in lab and rocket
frames

z coordinate of event is
same in lab and rocket
frames

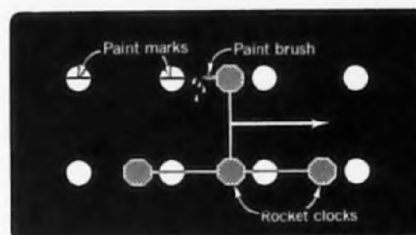


Fig. 12. Demonstration that y coordinate of an event is the same in laboratory and rocket frames.

tion of relative motion of the two frames. Equality of distance in each frame measured perpendicular to the direction of relative motion gives us a clean way to compare clocks in the two latticeworks. Let a flash of light bounce back and forth between a mirror mounted on the rocket reference clock and a mirror mounted on the $y = 1$ rocket clock directly above the reference clock. This flash will return to the origin every two meters of rocket light-travel time. We can trace the path of this flash of light in the laboratory frame up to the same y coordinate and back down again. Using the equality of the speed of light in the two frames we can calculate the laboratory time corresponding to the 2-meter round-trip time in the rocket frame. In the next section this study will lead to a demonstration of the invariance of the interval.

5. Invariance of the Interval

Looking for measure of separation AB that has same value in all inertial frames

Distance between two town gates is calculated from the difference between the x coordinates of the two gates and the difference between the y coordinates. How does one find the analogous physical quantity, the spacetime interval between two events? And between what two events shall this interval be evaluated?

Let event A be the *emission* of a flash of light. Let event B be the *reception* of this flash after its reflection from another object. All that matters in the end is the pair of events. Neither the light nor the object that reflects it is of any direct interest. Nevertheless, an analysis of the track of this pulse through spacetime reveals quickly and simply a quantity (the interval) that is associated with the two events and that has the same value in all inertial reference frames.

Event A : flash emission
Event B : flash reception

Event A : A spark plug fires. A flash of light flies up to reflector R in Fig. 13. Then the flash wings down. Event B : The flash is recorded. Now for the details (Figure 13).

Details: lab and rocket coordinates of events A and B

The spark plug fires in the laboratory frame at the zero of time and at the origin of the x, y, z coordinate system (crosshatched). The rocket passes by with such timing that it records the spark as taking place also at *its* origin (likewise crosshatched) and at *its* zero of time. So much for the coordinates of the event of emission:

$$\begin{aligned} x_{\text{emission}} &= 0, & y_{\text{emission}} &= 0, & t_{\text{emission}} &= 0 & (\text{laboratory frame}) \\ x'_{\text{emission}} &= 0, & y'_{\text{emission}} &= 0, & t'_{\text{emission}} &= 0 & (\text{rocket frame}) \end{aligned}$$

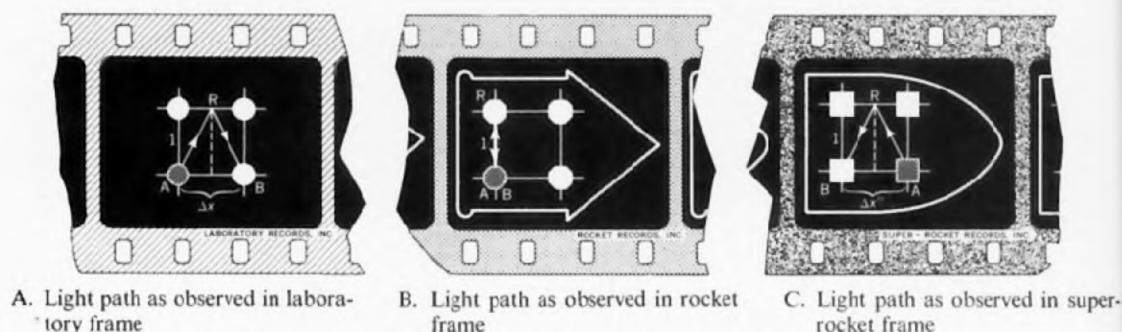


Fig. 13. Emission and reflection of the reference flash, and its reception at origin of rocket frame.

The reflector is origin.

The reception of emission. The light requires 2 meters in the rocket frame

$x'_{\text{reception}}$

More relevant than between the event of

In the laboratory, distance Δx to the low rocket speed, but the following the flash travels to $(\Delta x/2)$ and an alti

Recall that the speed of frame—the proper difference between the identical form

(4)

in meters of light-

Why is this time triangle in Fig. 13 conclusion that the two reference frames

Both time and space of emission are su

Table 5. Differences between event of

Laboratory

$$x_{\text{reception}} - x_{\text{emission}} =$$

$$t_{\text{reception}} - t_{\text{emission}} =$$

The time lapse is separation—just a town gates are different for the surveyors distance between g

(d)

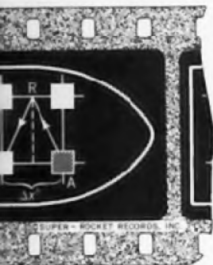
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laboratory frame)
rocket frame)



light path as observed in super-
rocket frame.
rocket frame.

The reflector is mounted on the rocket clock 1 meter directly above the origin.

The reception of the flash occurs in the rocket frame at the same place as the emission. The light flash travels a round-trip path of 2 meters. This trip requires 2 meters of light-travel time. The coordinates of the event of reception in the rocket frame are therefore

$$x'_{\text{reception}} = 0, \quad y'_{\text{reception}} = 0, \quad t'_{\text{reception}} = 2 \text{ meters}$$

More relevant than an absolute coordinate is the difference in coordinates between the event of reception and the event of emission

$$\begin{aligned} \Delta x' &= x'_{\text{reception}} - x'_{\text{emission}} = 0 \\ \Delta y' &= y'_{\text{reception}} - y'_{\text{emission}} = 0 \\ \Delta t' &= t'_{\text{reception}} - t'_{\text{emission}} = 2 \text{ meters} \end{aligned}$$

In the *laboratory* frame the light flash is received, not at the origin, but at the distance Δx to the right of the origin. High rocket speed implies a large Δx ; low rocket speed, a small Δx . (The distance is shown as 1 meter in the figure, but the following analysis is correct for any distance.) In the laboratory frame the flash travels the hypotenuse of two right triangles. Each has a base of $(\Delta x/2)$ and an altitude of 1 meter. The total length of the light path is

$$2[1 + (\Delta x/2)^2]^{1/2}$$

Recall that the speed of light is the same in the laboratory frame as in the rocket frame—the preposterous-but-true character of nature! Therefore the time difference between emission and reception in the laboratory frame is given by the identical formula

$$(4) \quad \Delta t = t_{\text{reception}} - t_{\text{emission}} = 2[1 + (\Delta x/2)^2]^{1/2}$$

in meters of light-travel time.

Why is this time greater than 2 meters? Because the hypotenuse of a right triangle in Fig. 13A, is greater than its altitude! There is no escape from the conclusion that *the time between emission and reception is not the same in the two reference frames.*

*Time between A and B
has different values for
lab and rocket
observers*

Both time and space differences between the event of reception and the event of emission are summarized in Table 5.

Table 5. Difference in coordinates between the event of reception and the event of emission.

Laboratory frame	Rocket frame
$x_{\text{reception}} - x_{\text{emission}} = \Delta x$	$x'_{\text{reception}} - x'_{\text{emission}} = \Delta x' = 0$
$t_{\text{reception}} - t_{\text{emission}} = \Delta t = 2[1 + (\Delta x/2)^2]^{1/2}$	$t'_{\text{reception}} - t'_{\text{emission}} = \Delta t' = 2 \text{ meters}$

The time lapse is different in the two reference frames; and so is the space separation—just as the coordinates Δx and Δy of the separation between two town gates are different for the Daytime and Nighttime surveyors! However, for the surveyors there was a combination of coordinates—the square of the *distance* between gates—that was the same for both of them

$$(\text{distance})^2 = (\Delta x)^2 + (\Delta y)^2 = (\Delta x')^2 + (\Delta y')^2$$

Interval between A and B has same value for lab and rocket observers

Is there any similar combination of coordinates for two events that will have the same value in the laboratory and rocket frames? Answer: Yes! the square of the interval

$$(5) \quad (\text{interval})^2 = (\Delta t)^2 - (\Delta x)^2 = (\Delta t')^2 - (\Delta x')^2 = (2 \text{ meters})^2$$

as one checks directly by substituting in the quantities listed in Table 5.

The rocket frame chosen in which to analyze these two events is a rather special one, in that both emission and reception occur at the same place in it. Figure 13,C, shows the path of the reflected light in a second rocket frame ("super-rocket frame") that is moving even faster relative to the laboratory frame than is the first rocket. In this second rocket frame the difference between the x coordinates for emission and reception (double primes on symbols) $x''_{\text{reception}} - x''_{\text{emission}} = \Delta x''$ is a *negative* quantity because the reception occurs on the negative x axis in this frame. Nevertheless, $(-\Delta x'')^2 = (\Delta x'')^2$ and we can still use the right triangles in Fig. 13,C, to show that the total length of the light path in this second rocket frame is given by the expression $2[1 + (\Delta x''/2)^2]^{1/2}$ —which is the same in form as that for the laboratory frame. The speed of light must have the same value in the second rocket frame as in the first rocket frame. Therefore the time between emission and reception is given by

$$t''_{\text{reception}} - t''_{\text{emission}} = \Delta t'' = 2[1 + (\Delta x''/2)^2]^{1/2}$$

Therefore

$$(\Delta t'')^2 - (\Delta x'')^2 = (2 \text{ meters})^2$$

also, and in summary

$$(6) \quad (\Delta t)^2 - (\Delta x)^2 = (\Delta t')^2 - (\Delta x')^2 = (\Delta t'')^2 - (\Delta x'')^2 = (2 \text{ meters})^2$$

Now forget the outgoing light flash, the reflector, and the returning light flash. They were only tools. They helped to identify the quantity that has the same value in different frames of reference. From now on focus on the quantity itself, the interval. Disregard the details of the derivation.

What has been learned? Two events, A and B, occur at the same point in the rocket frame ($\Delta x' = 0$) but at different times ($\Delta t' = 2 \text{ meters}$). Viewed in the laboratory frame, those same two events are separated in space by a distance Δx —the faster the rocket happens to be moving, the greater the distance. This result is hardly surprising. One is even entitled to say, "What could be more obvious!" The surprise comes elsewhere. First, *the time Δt between the two events as recorded in the laboratory frame does not have the same value as it has in the rocket frame.* Second, the time between A and B as punched out by the two relevant recording laboratory clocks is *greater* than the time between the same two events as recorded by the identical reference clock of the rocket: $\Delta t \geq \Delta t'$. Third, the factor of increase of the time (see Table 5)

$$\Delta t / \Delta t' = [1 + (\Delta x/2)^2]^{1/2}$$

is close to unity (that is, the increase itself is very small) if the distance Δx covered by the rocket between events A and B is small. However, if the rocket moves very fast, Δx is a very great quantity, and the factor of discrepancy be-

tween the two times in *time* as recorded in the laboratory frame is the difference between the two times in the rocket frame ($\Delta x \neq \Delta x'$)

The rocket speed is also very large. The size of Δx . In consequence, $(2 \text{ meters})^2$, no matter

All of these relations use of the first principle (Δx) has a standard value, is a magnitude (1 meter). What then was the value of $(\Delta x)^2$ has the value of the keystone was the difference in the laws of physics. This principle was to reason that distance recorded as of event frame. Otherwise with the shorter was employed to laboratory frame Kennedy-Thorndike light-travel path is longer than the side two triangles: upon the laboratory frame

In brief, in the great ideas that distance, invariant coordinates upon the

If Fig. 13,A, the member, the foreground seems to be for the lapse of time the rocket? Has it the direction of motion distinguished from "distances"? What all Does not this differ

Interval AB has same value in all rocket frames!

What is the same in two inertial frames; what is nearly the same; and what is different

two events that will have
answer: Yes! the square

(2 meters)²

listed in Table 5.

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$(\Delta x''/2)^2$

$(\Delta x'')^2 = (2 \text{ meters})^2$

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factor of discrepancy be-

tween the two times is enormous. Fourth, despite this newly found difference in *time* as recorded in the two reference frames, and despite the long-known difference between the *space* separation of the events in the two reference frames ($\Delta x \neq \Delta x' = 0$), there is nevertheless a quantity that *is* the same in the laboratory frame as the 2 meters of elapsed light-travel time between A and B in the rocket frame. This quantity is the interval,

$$(\text{interval}) = [(\Delta t)^2 - (\Delta x)^2]^{1/2}$$

The rocket speed may be very high. Then Δx will be very large. But then Δt is also very large. Moreover, the magnitude of Δt is perfectly tailored to the size of Δx . In consequence, the special quantity $(\Delta t)^2 - (\Delta x)^2$ has the value (2 meters)², no matter how great Δx and Δt individually may be.

All of these relationships can be seen at a glance in Fig. 13,A. The hypotenuse of the first right triangle is $\Delta t/2$. Its base is $\Delta x/2$. To say that $(\Delta t)^2 - (\Delta x)^2$ has a standard value, and thus to state that $(\Delta t/2)^2 - (\Delta x/2)^2$ has a standard value, is to say that the altitude of this right triangle has a fixed magnitude (1 meter in the diagram) no matter how fast the rocket is going. What then was the keystone of the argument establishing the fact that $(\Delta t)^2 - (\Delta x)^2$ has the value (2 meters)², no matter how fast the rocket is moving? The keystone was the principle of relativity, according to which there is no difference in the laws of physics between one inertial reference frame and another. This principle was put to use here in two very different ways. First, it was used to reason that distances at right angles to the direction of relative motion are recorded as of equal magnitude in the laboratory frame and in the rocket frame. Otherwise one frame could be distinguished from the other as the one with the shorter perpendicular distances. Second, the principle of relativity was employed to deduce that the speed of light must be the same in the laboratory frame as in the rocket frame—a deduction supported by the Kennedy-Thorndike experiment. The speed being the same, the fact that the light-travel path in the laboratory frame (the hypotenuses of two triangles) is longer than the simple round-trip path in the rocket frame (the altitudes of the two triangles: up 1 meter and down again) directly implies a longer *time* in the laboratory frame than in the rocket frame.

In brief, in the one elementary triangle of Fig. 13,A, are displayed the four great ideas that underlie all of special relativity: invariance of perpendicular distance, invariance of the speed of light, dependence of space *and* time coordinates upon the frame of reference, and invariance of the interval.

If Fig. 13,A, thus epitomizes all of special relativity in a form easy to remember, the foregoing analysis of the figure nevertheless leads to what at first sight seems to be a preposterous conclusion. How can it possibly make sense for the lapse of time between two events to be longer in the laboratory than in the rocket? Has it not already been argued that "distances at right angles to the direction of relative motion" are equal, "otherwise one frame could be distinguished from the other as the one with the shorter perpendicular distances"? What about the difference between *time* lapses in the two frames? Does not this difference give a way to differentiate physics in one frame from

*One diagram illustrates
the four great ideas of
special relativity*

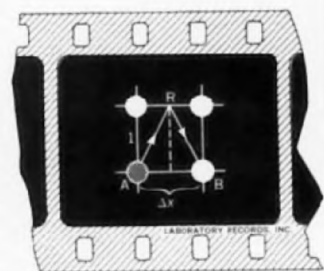


FIG. 13,A, PAGE 22

*Is inequality of lab and
rocket time lapses a
paradox?*

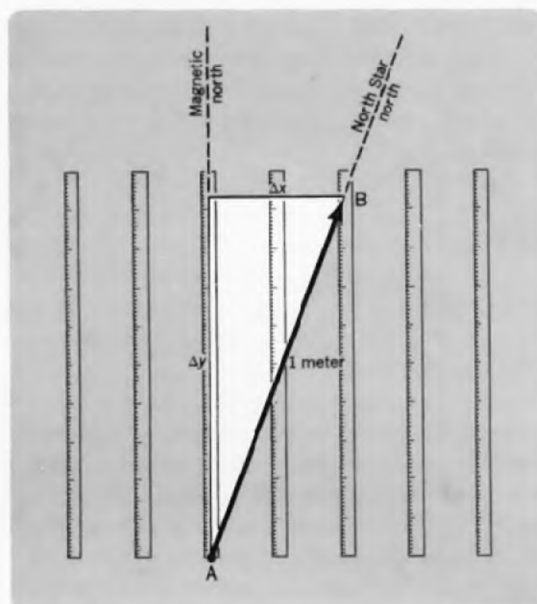


Fig. 14. The north-south separation of point B from point A (the "northing of B relative to A") depends upon the choice of the direction north.

*Relativity of time
(Lorentz) compared
with relativity of
"northing" (Euclid)*

physics in the other? Yet is not such a difference ruled out by the principle of relativity—the principle that one inertial reference frame is as good as another?

For answers to these questions, turn back to the parable of the surveyors. Consider point B in Fig. 14. It is one meter straight north of another point, A, according to the reckoning of the Nighttime surveyor and his North Star north. Now consider the location of point B according to the Daytime surveyor and his magnetic north. Is the y separation Δy between A and B (surveying terminology: the *northing* of B relative to A) also one meter in the Daytime frame? No, Δy is *less* than one meter! How can this be? Because the altitude (Δy) of a right triangle is shorter than the hypotenuse (1 meter). Does this mean that the rules of surveying in the Daytime coordinate system are different from those in the Nighttime coordinate system? Evidently not! Similarly, there is no flaw in the construction or functioning of the laboratory clocks that makes them give longer readings for the time lapse AB. The "discrepancy" between the laboratory clocks and the rocket clock is caused instead by the character of spacetime geometry itself. That is the way the world is built! The analogy between the Lorentz geometry of spacetime and the Euclidean geometry of the surveyors' world is expanded in Table 6 (pages 28 and 29).

6. The Spacetime Diagram; World Lines

*The spacetime
diagram: simple way
to display events*

A simple way to look at the events of emission and reception of the last section is to plot the position of the event on the horizontal axis and the time of the event on the vertical axis of a *spacetime diagram* (Fig. 15). The light is *emitted* from a spark plug attached to the reference clock of the first rocket. This plug fires at the instant when this clock passes the laboratory reference

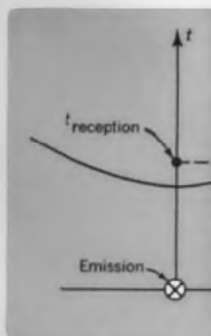
clock. Both clocks
at the origin of the

It is also located at
the observer

The further his
spacetime diagram
the *reception* of the
than the reference

as already recorded
laboratory frame

as shown in Fig. 1
faster than the first
(Fig. 15,C)



A. Laboratory spacetime

Fig. 15. Spacetime diagram showing reflection. The hyperbola is defined by the equation $t'^2 - x'^2 = t''^2 - x''^2$.

clock. Both clocks then read zero. Therefore the event of emission is located at the origin of the spacetime diagram plotted by the rocket observer

$$x'_{\text{emission}} = 0, \quad t'_{\text{emission}} = 0$$

It is also located at the origin of the spacetime diagram made by the laboratory observer

$$x_{\text{emission}} = 0, \quad t_{\text{emission}} = 0$$

The further history of the relevant light ray looks different as plotted in the spacetime diagrams of the laboratory and the two rockets. In the first rocket the *reception* of the reflected flash occurs at $x' = 0$ and 2 meters of time later than the reference event

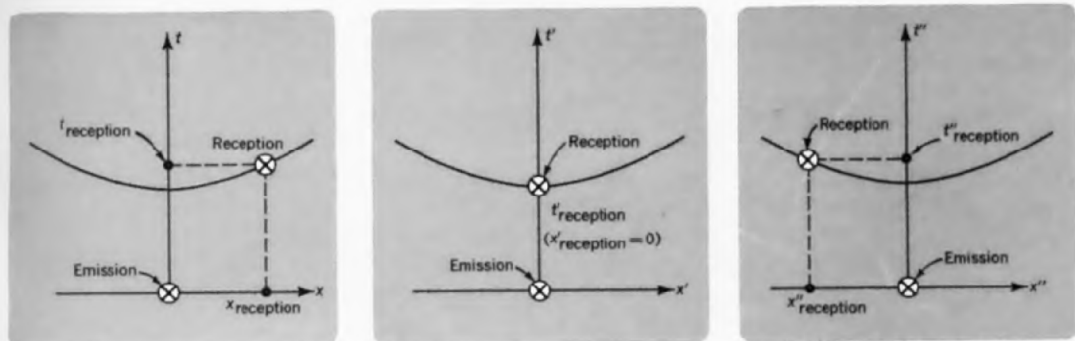
$$x'_{\text{reception}} = 0, \quad t'_{\text{reception}} = 2 \text{ meters}$$

as already recorded in Table 5 and as seen more directly in Fig. 15,B. In the laboratory frame the event of reception is located to the right of the origin

$$\begin{aligned} x_{\text{reception}} &= \text{a positive quantity} \\ t_{\text{reception}} &= [(2 \text{ meters})^2 + x_{\text{reception}}^2]^{1/2} \\ &= \text{a time greater than 2 meters} \end{aligned}$$

as shown in Fig. 15,A. In the second rocket frame (the second rocket is moving faster than the first!) the event of reception appears to the left of the origin (Fig. 15,C)

$$\begin{aligned} x''_{\text{reception}} &= \text{a negative quantity} \\ t''_{\text{reception}} &= [(2 \text{ meters})^2 + (x''_{\text{reception}})^2]^{1/2} \\ &= \text{a time greater than 2 meters (again!)} \end{aligned}$$



A. Laboratory spacetime diagram

B. Rocket spacetime diagram

C. Super-rocket spacetime diagram

Fig. 15. Spacetime diagrams showing emission of the reference flash and its reception after reflection. The hyperbola drawn in each figure satisfies the equations $(\text{interval})^2 = t^2 - x^2 = t'^2 - x'^2 = t''^2 - x''^2$

Table 6. Comparison of the difference in northing between points A and B in Daytime and Nighttime coordinate systems and the difference in time between events A and B in laboratory and in rocket reference frames.

Questions	Answers of a student of surveying concerning difference in northing between points A and B. (See Fig. 14.)	Answers of a student of spacetime physics concerning difference in time between events A and B. (See Fig. 13.)
In which frame of reference does the separation of B from A appear simplest?	Coordinate system of Nighttime surveyor based on North-Star north.	Reference frame of rocket.
What is simplifying feature in this frame of reference?	Points have same x' coordinate; or $\Delta x' = 0$.	Events have same x' coordinate; or $\Delta x' = 0$.
Why does this feature simplify the measurement of the separation AB?	One meter stick oriented to North-Star north suffices (1) to verify that both points <i>do</i> have the same x' coordinate and (2) to measure directly the northing of B relative to A.	One recording clock attached to rocket frame suffices (1) to verify that both events <i>do</i> have the same x' coordinate and (2) to measure directly the time delay of B relative to A.
What is an alternative frame for analyzing the separation AB?	Coordinate system of Daytime surveyor based on magnetic north.	Reference frame of laboratory.
What complication is there in analyzing the separation in this alternative frame?	No single one of <i>his</i> meter sticks, oriented to magnetic north, can locate both A and B.	No single one of the recording laboratory clocks can register both A and B.
How is this difficulty met?	<i>Two</i> of these north-oriented meter sticks are needed, one located Δx meters to the right of the other.	<i>Two</i> of these laboratory clocks are needed, one located Δx meters to the right of the other.
What is the reading on the first of these measuring devices?	Point A at $y = 0$.	Event A at $t = 0$.
And the reading on the second of these measuring devices?	Point B located Δy meters north.	Event B delayed by Δt seconds.
Does the coordinate thus found for B directly measure its separation from A?	No! The northing Δy is less than the distance AB. More precisely, $\Delta y = [(AB)^2 - (\Delta x)^2]^{1/2}.$	No! The delay Δt is greater than the interval AB. More precisely, $\Delta t = [(AB)^2 + (\Delta x)^2]^{1/2}.$
Then how does one find the separation AB from measurements in this frame of reference?	From the formula for distance, $(\text{distance})^2 = (\Delta x)^2 + (\Delta y)^2.$ (Test by substituting in the expression for Δy from the entry above!)	From the formula for interval, $(\text{interval})^2 = (\Delta t)^2 - (\Delta x)^2$ (Test by substituting in the expression for Δt from the entry above!)

What is the distinction between present examples of measurements made in primed and the unprimed frames?

Isn't there something postmodern about this?

Yes! Does not this analogy prove that there is an inner contradiction in reasoning?

Is there some fundamental difference between the primed and the unprimed frames of reference that is responsible for the one-sided difference between coordinate systems?

Then what *is* responsible for this one-sidedness?

How can the identical character of the physics in the two frames of reference be illustrated?

For such a choice of events, what is the distinction between measurements made in primed and the unprimed frames?

How can the discussion be summarized?

g between points A and B
systems and the difference in
ory and in rocket reference

*Answers of a student of
spacetime physics concerning
difference in time
between events A and B.
(See Fig. 13.)*

Reference frame of rocket.

Events have same x' coordi-
nate; or $\Delta x' = 0$.

One recording clock attached
to rocket frame suffices (1) to
verify that both events *do*
have the same x' coordinate
and (2) to measure directly
the time delay of B relative
to A.

Reference frame of labora-
tory.

No single one of the record-
ing laboratory clocks can
register both A and B.

Two of these laboratory
clocks are needed, one lo-
cated Δx meters to the right
of the other.

Event A at $t = 0$.

Event B delayed by Δt sec-
onds.

No! The delay Δt is greater
than the interval AB. More
precisely,

$$\Delta t = [(AB)^2 + (\Delta x)^2]^{1/2}.$$

From the formula for inter-
val,

$(\text{interval})^2 = (\Delta t)^2 - (\Delta x)^2$
(Test by substituting in the
expression for Δt from the
entry above!)

What is the distinction in the
present examples between
measurements made in the
primed and the unprimed
frames?

Isn't there something pre-
posterous about this result?

Yes! Does not this discrep-
ancy prove that there is some
inner contradiction in the
reasoning?

Is there some fundamental
difference between the primed
and the unprimed frame of
reference that is responsible
for the one-sided difference
between coordinate values?

Then what *is* responsible for
this one-sidedness?

How can the identical char-
acter of the physics in the two
frames of reference be readily
illustrated?

For such a choice of C, what
is the distinction between
measurements made in the
primed and the unprimed
frames?

How can the discussion be
summarized?

Δy is less than $\Delta y'$ ($= AB$).

Meaning that identical meter
sticks give nonidentical
northings?

No! A single Nighttime meter
stick suffices to establish the
distance AB. But there is no
single Daytime meter stick
with which one establishes
the (lesser) magnetic northing
of B relative to A. Therefore
no Daytime meter stick can
be said to disagree with the
Nighttime meter stick.

For $\Delta y < \Delta y'$? No!

The point B happens to lie
on the same North-Star north
line as A, but *not* on the same
magnetic north line as A.

Pick a point C that has the
same x coordinate as A (C in
the line of magnetic north
relative to A)

Δy ($= AC$) is greater than
 $\Delta y'$.

There is no paradox about
northward component of AB
having different values in two
coordinate systems; the dis-
crepancy is not a fault of the
meter sticks; not even a fault
at all; the "discrepancy" is
caused by the inner workings
of Euclidean geometry.

Δt is greater than $\Delta t'$ ($= AB$).

Meaning that identical clocks
record nonidentical times?

No! A single rocket clock
records the interval AB. But
there is no *single* laboratory
clock with which one estab-
lishes the (greater) laboratory
time delay of B relative to A.
Therefore no laboratory
clock can be said to disagree
with the rocket clock.

For $\Delta t > \Delta t'$? No!

Event B by chance occurs at
the same point in the rocket
frame as A, but *not* at the
same point as A in the labo-
ratory frame.

Pick an event C that has the
same x coordinate as A (C
at the same *place* as A in the
laboratory frame, but later
in *time*).

Δt ($= AC$) is less than $\Delta t'$.

There is no paradox about
time lapse from A to B hav-
ing different values in two
reference frames; the dis-
crepancy is not a fault of the
clocks; not even a fault at all;
the "discrepancy" is caused
by the very structure of the
geometry of the spacetime in
which all physics takes place.

*Invariant interval
corresponds to
hyperbola on
spacetime diagram*

The different points marked "reception" in the different spacetime diagrams all refer to the *same event*. The event is the same but its coordinates in different frames are different. What do these different coordinates of the same event have in common? They all satisfy the equation

$$(\text{time separation})^2 - (\text{space separation})^2 = (\text{interval})^2 = \text{constant}$$

This is the equation of a *hyperbola*. Therefore, *an event which is plotted on the hyperbola $t^2 - x^2 = (\text{constant})$ in the spacetime diagram of any laboratory or rocket frame will be plotted somewhere on a hyperbola with the same equation in the spacetime diagram of every other laboratory and rocket frame.*

Is there likewise a single curve that correlates the different coordinate values obtained by the Daytime and Nighttime surveyors for a single gate? The x and y coordinates of, say, gate A with respect to the town square depends on the choice of the north direction (Fig. 16). The Daytime and Nighttime plots of this gate are shown in Fig. 17, parts A and B. Think of a third and still different set of axes rotated even more than the Nighttime axes relative to the Daytime axes. For the surveyor who uses this third set of axes the x'' coordinate of gate A may be negative (Fig. 17,C).

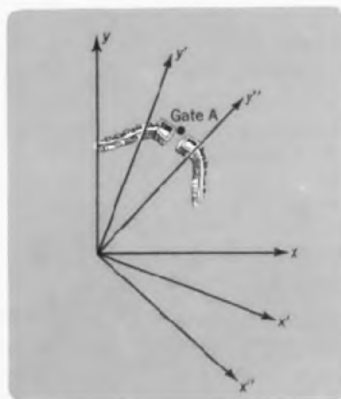
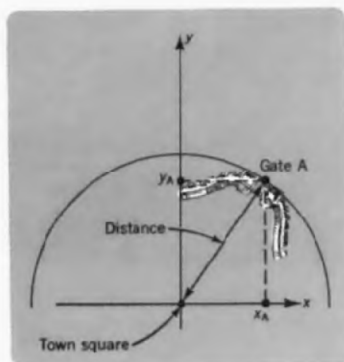
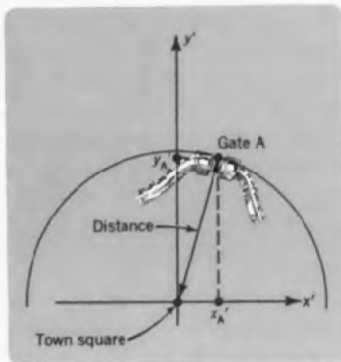


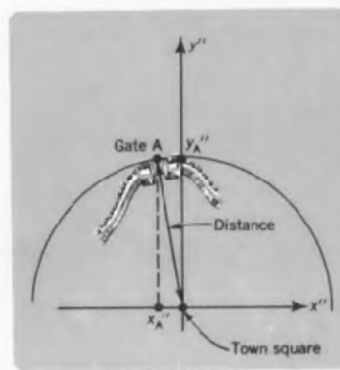
Fig. 16. Relative standards of north for Daytime, Nighttime, and a third surveyor respectively.



A. Daytime plot



B. Nighttime plot



C. Plot by third surveyor

Fig. 17. Coordinates of Gate A as observed by Daytime, Nighttime, and a third surveyor respectively. The circle drawn in each figure satisfies the equations $(\text{distance})^2 = x^2 + y^2 = x'^2 + y'^2 = x''^2 + y''^2$

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same gate. The ga
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They all satisfy

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timelike, lightlike
positive, zero, or

Table 7. Classif

	D
Time part of interval	
Time part of interval	
Space part of interval	

The value of t
symbols depend
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(7)

erent spacetime diagrams
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interval)² = constant

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Daytime and Nighttime
. Think of a third and still
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set of axes the x'' coordi-

Relative standards of north
me, Nighttime, and a third
respectively.



by third surveyor

eyor
 $y^2 =$

The different points marked "gate A" in the different plots all refer to the *same gate*. The gate is the same but its coordinates in different plots are different. What do these different coordinates of the same gate have in common? They all satisfy the equation

$$(x \text{ separation})^2 + (y \text{ separation})^2 = (\text{distance})^2 = \text{constant}$$

This is the equation of a *circle*. Therefore, a point that is plotted on the circle $x^2 + y^2 = (\text{constant})$ in the coordinate system of any surveyor will be plotted somewhere on a circle with the same equation in the coordinate system of every other surveyor.

Here is the fundamental difference between textbook Euclidean geometry and the real Lorentz geometry of spacetime. In Euclidean geometry the *distance* between two points is an invariant, and as a result, for all surveyors gate A will lie somewhere in the xy plane on a *circle* centered on the town square. In Lorentz geometry the *interval* between events is an invariant, and as a result, for all laboratory and rocket observers a given event will lie somewhere on a *hyperbola* in the spacetime diagram when referred to the reference event.

In Euclidean geometry the length—or its square—is always a *positive* quantity

$$(\Delta x)^2 + (\Delta y)^2 = (\Delta x')^2 + (\Delta y')^2 \geq 0$$

In contrast, the squared interval of Lorentz geometry

$$(\Delta t)^2 - (\Delta x)^2 = (\Delta t')^2 - (\Delta x')^2$$

may be *positive*, *negative*, or *zero*, depending on whether the time or the space component predominates. Moreover, whichever of these three descriptions characterizes the interval in one reference frame also characterizes the interval in *any other* reference frame, because the interval has the same value in all frames. Accordingly we find that nature provides a fundamental way to classify the relation between two events. An interval between two events is called *timelike*, *lightlike*, or *spacelike* depending on whether the squared interval is positive, zero, or negative, respectively, as shown in Table 7.

*Invariant distance
corresponds to circle
on xy diagram*

*Three types of
separation between
two events:
timelike
lightlike
spacelike*

Table 7. Classification of the relation between two events.

Description	Squared interval	Name
Time part of interval dominates over space part	positive	timelike interval
Time part of interval equals space part	zero	lightlike interval
Space part of interval dominates over time part	negative	spacelike interval

The value of the interval between two events is represented by different symbols depending on whether it is timelike or spacelike. The value of a timelike interval is given the Greek letter tau (τ) and is called the *invariant timelike distance* between two events or the *proper time* (or sometimes the *local time*) between the two events

$$(7) \quad \Delta\tau = [(\Delta t)^2 - (\Delta x)^2]^{1/2}$$

*Proper time and proper
distance*

The value of a spacelike interval between two events is given the Greek letter sigma (σ) and is called the *invariant spacelike distance* or the *proper distance* between the two events

$$(8) \quad \Delta\sigma = [(\Delta x)^2 - (\Delta t)^2]^{1/2}$$

World line of particle

Figure 18 represents as a function of time the location of a particle which started along the x axis from the origin at $t = 0$. Such a plot of position versus time in a spacetime diagram is called the *world line* of the particle. Each lattice clock encountered by the particle punches out the time of coincidence. Thus the world line of the particle can be considered to be made up of these separate events of coincidence. No particle has ever been observed to travel faster than light. Therefore a particle will always travel *less* than one meter of distance in one meter of light-travel time. It follows that events along this world line will have a greater time separation than their separation in space: the world line of a particle will consist of events that are *timelike* with

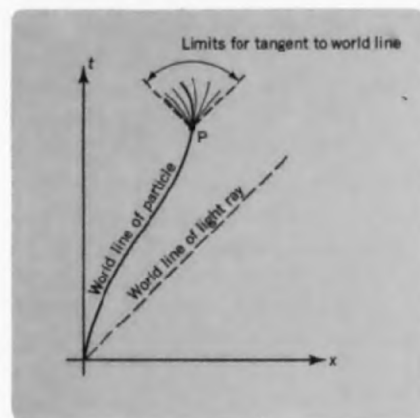


Fig. 18. Timelike world line of a particle.

respect to the initial event and to each other. In other words, *a particle must follow a timelike world line*. A timelike world line is characterized at every point P by a local tangent which lies between the world lines of light rays originating at that point. These light rays travel one meter of distance in one meter of light-travel time. Events along the world line of a light ray have equal space and time separations. Therefore the world line of a light ray consists of events that are *lightlike* with respect to the initial event and to each other. In other words *light rays follow lightlike world lines*.

Path in space has length

Distance is a central idea in all applications of Euclidean geometry. For instance, using a flexible tape measure it is easy to measure the distance s along a path that starts at the town square and winds out through gate A (Fig. 19,A). The distance Δs between any two nearby points on the path (for instance, those marked 3 and 4 in the figure) can also be *calculated* using the difference in coordinates Δx and Δy of the two points with respect to any coordinate system. Since distance is invariant, the distance between these two points will be the same when calculated in any coordinate system even though the separate coordinates Δx and Δy have different values in different coordinate systems. Elsewhere along the path the distance between another pair of nearby points will also be independent of the coordinate system used in evalu-

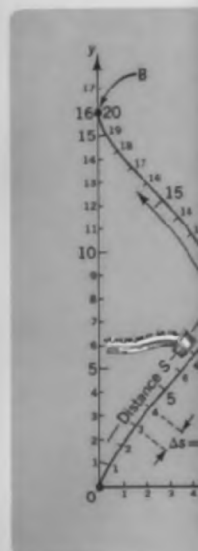


Fig. 19. A. Distance which starts at the town square and winds out through gate A. The total distance along the path from point O to point B is the same as the distance along the straight line from point O to point B.

ating that distance. The path is not a straight path! Thus difference in distance is not the same as the distance from the final point B.

It is possible to measure distance in Euclidean geometry, for example, along the path of a particle. The path is evidently not a straight line, so the length of different segments of the path is well known as the distance. In Euclidean geometry, the distance between two points is less than a straight line. The difference of length is the distance between the two points, a tape measure for distance.

Proper time is the time interval between two events in Euclidean geometry. For instance, an event B. Then the proper time is the time interval between the two events. The lapse of proper time is the time interval between the two events, world line and a straight line. The proper time is the time interval between the two events, closely at how the time interval is measured.

Consider a particle moving with a constant speed. Let the particle be a clock carried by the particle. The particle flashes (for instance, a light flash) using the difference in time interval between the two events.

ts is given the Greek letter
ance or the *proper distance*

ocation of a particle which
Such a plot of position
world line of the particle.
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nsidered to be made up of
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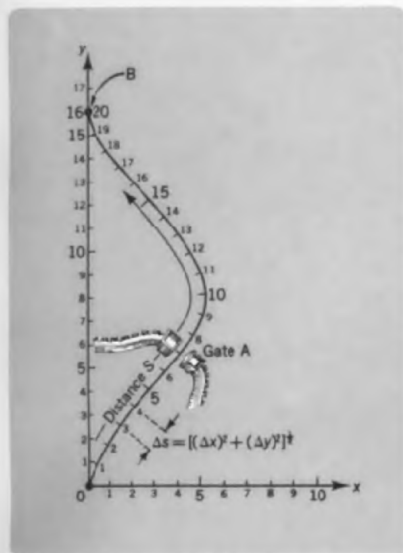


Fig. 19, A. Distance along a winding path which starts at the town square. Notice that the total *distance along the winding path* from point O to point B is *greater* than the *distance along the straight y axis* from point O to point B.

ating that distance. So too for the *sum* of the lengths of all the segments of the path! Thus different surveyors using different coordinate systems will all agree on the distance along a given path from a specified initial point O to a specified final point B.

It is possible to proceed from O to B along quite another path—for example, along the straight line OB in Fig. 19, A. The length of this alternative path is evidently different from that of the original path. This difference in length of different paths between O and B is a feature of Euclidean geometry so well known as to occasion hardly any comment and certainly no surprise. In Euclidean geometry a curved path between two specified points is *longer* than a straight path between the same two points. The existence of the difference of length between two paths violates no law. No one would claim that a tape measure fails to perform properly when laid along a curved path.

Proper time is to a world line in Lorentz geometry what length is to a path in Euclidean geometry. The world line is started at an event O and ended at an event B. There are many different world lines that start at O and end at B. The lapse of proper time on each is well defined; but it differs between one world line and another. Is this surprising? Then it is appropriate to look more closely at how the proper time is defined and measured.

Consider a particle moving from O to B along the curved world line of Fig. 19, B. In this example, the particle travels along the *x* axis at a changing speed. Let the particle emit a flash of light every meter of time as recorded on a clock carried with it. The proper time $\Delta\tau$ between any two consecutive flashes (for instance, those marked 3 and 4 in the figure) can be *calculated* using the difference in coordinates Δx and Δt of the two events measured in a

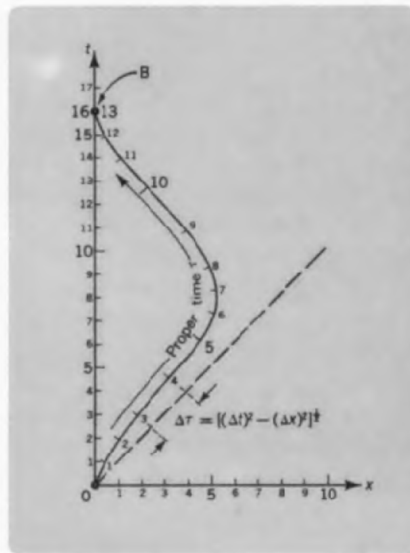


Fig. 19, B. Proper time along a curved world line in a spacetime diagram. Notice that the total *proper time along the curved world line* from event O to event B is *smaller* than the *proper time along the straight t axis* from event O to event B.

Length is shortest for direct path

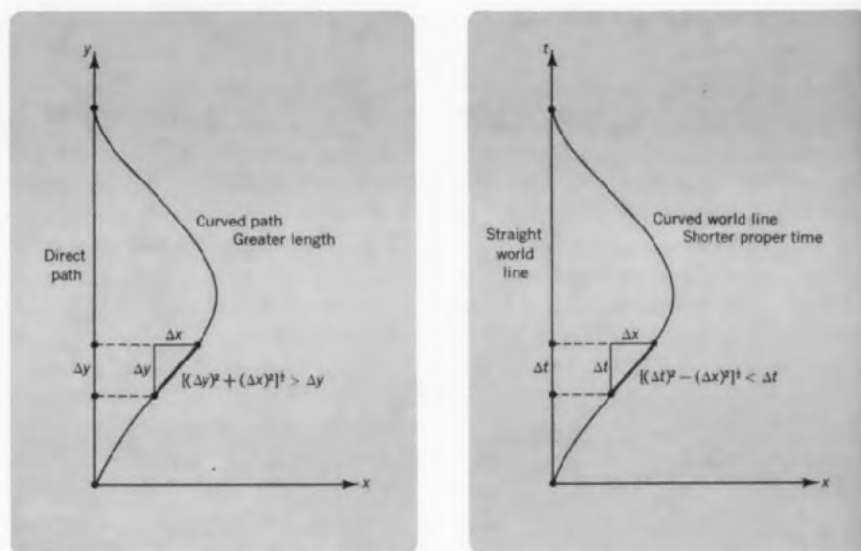
Stretch of world line measured by proper time

particular inertial frame. Because the interval is invariant, the proper time between these two events will be the same when calculated in any inertial frame, even though the separate space and time coordinates Δx and Δt will have different values in different reference frames. Elsewhere along the world line the interval between another pair of consecutive flashes will also be independent of the reference frame which is used in evaluating that interval. So too for the *sum* of the proper-time intervals of all the flashes along the world line! Thus observers in different inertial reference frames will all agree on the proper time along a given world line from a specified initial event O to a specified final event B.

Proper time is longest
for direct world line

It is possible to proceed from event O to event B along quite another world line—for example, along the straight world line OB in Fig. 19,B. The elapsed proper time along this alternative world line is different from the proper time along the original world line. In Lorentz geometry a curved world line between two specified events is *shorter* than the direct world line between the same two events—shorter as measured by the elapsed proper time along the world line. This contrast between Euclidean and Lorentz geometry is shown in Fig. 20. The *distance* between nearby points along a *curved path* is always equal to or *greater* than the *y* displacement between those two points. In contrast, the *proper time* between nearby events along a curved *world line* is always equal to or *less* than the corresponding time along the direct world line. The determination of proper time is a fundamental method of comparing different world lines between two events.

The change of slope of the world line from point to point in Fig. 19,B, and Fig. 20,B, means that the clock being carried along the world line changes velocity: it is *accelerated*. Different clocks will behave differently when accelerated unless these clocks are sufficiently small. As a rule a clock can withstand a great acceleration only if it is small



A. Euclidean geometry

B. Lorentz geometry

Fig. 20. Contrast between Euclidean and Lorentz geometry. In Lorentz geometry the *curved* world line is traversed in the *shorter* proper time.

Fig. 21. Three alternative world lines between event O and event B. The world line OQB is the ideal limit of straight world lines as the speed at events Q approaches the speed of light.

and compact. The sharper the curves of the world line, the shorter the proper time we assume the ideal limit of straight world lines.

We are now free to proceed from event O to event B along quite another world line—for example, along the straight world line OB in Fig. 19,B. That we slowed down. Now (force!); also make proportion of time. Thus come eventualities are too short (OQB in Fig. 21). In by (1) the initial event point Q, halfway in the lapse of proper point—and thus to

Path OPB is the proper time from O to B in an inertial reference system.

In contrast, on the curved world line segment the space

$\tau_{ORB} = (\text{twice})$

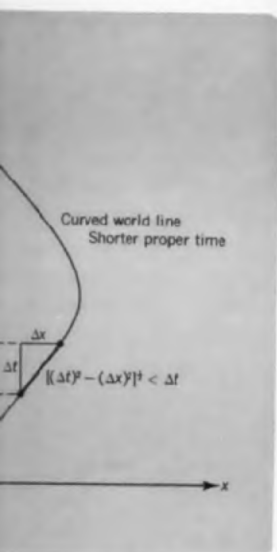
Of course no clock is not actually attainable. Or, in other words, light, and yet less than the other will short as one please.

As distinguished of proper time

invariant, the proper time calculated in any inertial coordinates Δx and Δt will be the same. Elsewhere along the world line, the same flashes will also be in the same order. So the flashes along the world line in all frames will all agree on the time interval from the specified initial event O to a

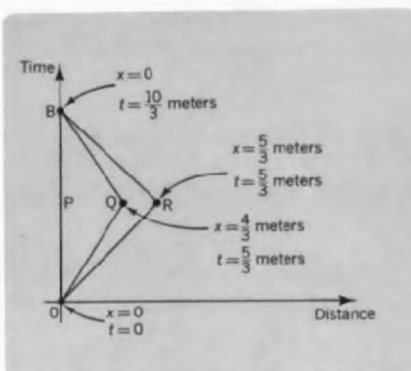
along quite another world line B in Fig. 19,B. The elapsed proper time is different from the proper time along a curved world line between the two events. The proper time along the curved world line is shown in Fig. 19,B. The elapsed proper time along the curved world line is always less than the proper time along the direct world line. The proper time along the direct world line is the proper time along the direct world line. The proper time along the direct world line is the proper time along the direct world line.

In Fig. 19,B, and Fig. 20,B, the world line is curved. It is accelerated unless these clocks are in free fall. Acceleration only if it is small



geometry
in Lorentz geometry the curved

Fig. 21. Three alternative world lines connecting event O and event B. The sharp changes of speed at events Q and R have been drawn for the ideal limit of small (acceleration-proof) clocks.



and compact. The smaller the clock, the more acceleration it can withstand, and the sharper the curves on the world line can be. In all figures like Fig. 19,B, and Fig. 20,B, we assume the ideal limit of infinitesimally small clocks.

We are now free to analyze a motion in which the particle and the clock are subject to a great acceleration. In particular, consider a simple special case of the world line of Fig. 19,B. That world line gradually changed in slope as the particle speeded up and slowed down. Now make the period of speeding up shorter and shorter (great driving force!); also make the period of slowing down shorter and shorter. In this way the proportion of time spent in steady motion at high speed becomes greater and greater. Thus come eventually to the limiting case where the times of acceleration and deceleration are too short even to show up on the scale of the spacetime diagram (world line OQB in Fig. 21). In this simple limiting case the whole history of the motion is specified by (1) the initial event O, (2) the final event B, and (3) the coordinate x of the turnaround point Q, halfway in time between O and B. In this case it is particularly easy to see how the lapse of proper time between O and B depends upon the coordinate x of the halfway point—and thus to compare the three world lines OPB, OQB, and ORB.

Path OPB is the world line of a particle that does not move: $x = 0$ for all time. The proper time from O to B by way of P is evidently equal to the time as measured in the inertial reference system.

$$\tau_{OPB} = 10/3 \text{ meters of light-travel time}$$

In contrast, on the way from O to B by way of R, each stretch is lightlike: for each segment the space and time components of the displacement are equal, and

$$\tau_{ORB} = (\text{twice proper time on stretch OR}) = 2 [(t)^2 - (d)^2]^{1/2} = 0$$

Of course no clock can travel as fast as the speed of light. Therefore the world line ORB is not actually attainable. However, it is the ideal limit of world lines that actually are attainable. Or, in other words, one can find a speed sufficiently close to the speed of light, and yet less than the speed of light, so that a trip with this speed first one way then the other will bring an ideal clock back to $x = 0$ with a lapse of proper time as short as one pleases.

As distinguished from the limiting case ORB, the world line OQB demands an amount of proper time

$$\begin{aligned} \tau_{OQB} &= (\text{twice proper time on stretch OQ}) \\ &= 2 [(5/3)^2 - (4/3)^2]^{1/2} \\ &= 2 \left[\frac{25 - 16}{9} \right]^{1/2} \\ &= 2 \text{ meters of light-travel time} \end{aligned}$$

Proper time from O to B compared for three world lines

This is *less* proper time than the proper time $\tau_{OPB} = 3 \frac{1}{3}$ meters that characterized the "direct" world line OPB!

Evidently *proper* time in the real physical world of spacetime differs remarkably from the *distance* of textbook Euclidean geometry. Distance is shortest for the direct route: "A straight line is the shortest distance between two points." In contrast, the lapse of proper time is less for the traveler who travels away, accelerating to high speed, then reverses his course and comes back, than for the man who stays home! (See Ex. 27 and Ex. 49 on the clock paradox). In brief, proper time is the appropriate measure of time as it will be observed by a particle that travels along a world line, just as the graduations along a flexible tape provide the appropriate measure of distance covered by a traveler along a winding path.

7. Regions of Spacetime

*Interval including y
and z coordinates*

Thus far in dealing with the interval between two events, A and B, we have had occasion to consider only the situation in which they have the same y and z coordinates. In this situation the separation in space between the two events is measured by the single quantity

$$\text{distance} = \Delta x$$

The interval is given by the expression

$$[(\Delta t)^2 - (\Delta x)^2]^{1/2}$$

However, the orientation of the x, y, and z coordinate axes is evidently a matter of arbitrary choice. With another orientation of the axes the component Δx of the separation between the two events will ordinarily have quite a different value. Yet the separation in space between the two events is quite independent of any choice of orientation, and is given by the expression

$$(\text{distance})^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

In other words, this is the quantity that must replace $(\Delta x)^2$ in the full formula for the interval. Thus we have the complete expression for the interval between two events

$$A \quad \text{at} \quad (t, x, y, z)$$

and

$$B \quad \text{at} \quad (t + \Delta t, x + \Delta x, y + \Delta y, z + \Delta z)$$

in the form

$$(9) \quad (\text{interval of proper time})^2 = (\text{time})^2 - (\text{distance})^2 \\ = (\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

when the interval is timelike; and when it is spacelike,

$$(10) \quad (\text{interval of proper distance})^2 = (\text{distance})^2 - (\text{time})^2 \\ = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (\Delta t)^2$$

How is one to understand the new kind of geometry described by an expression for "interval of proper distance" that contains three plus signs—as in ordinary Euclidean geometry—but also one minus sign? One can follow

Minkowski (1908) defined by

or

(11)

Then the expression

(interval of

The signs are now Euclid—in four dimensions. In the formula, Minkowski adds time by itself, and not the kind of union of time and space. The union of space and time, stars, atoms, and particles, is different for different observers. The interval is the same for every observer.

Minkowski's insight focuses attention on the fact that all frames of reference are equivalent as velocity, energy, and momentum reference.

Today we have the right to say that time is not wrong to say that time is not time measured by the surveyor. The rates of the surveyor are not the x and t of one another? How can they be equal footing, as in the like interval? Equal to this formula that marks the difference between the move this minus sign would if w were a number seconds, or $(-1)^{1/2}$ example. Consequently the distance term $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ has two signs the same.

The difference in the expression for the interval is quite different. It is never possible to have less all three of the contrast, the interval

†A. Einstein *et al.*, *The*