Thus, $A/(4\pi R^2) = 7.7 \times 10^{-45}$, and $N = 12$ photons/s. Since the star is barely visible, we may conclude that 12 photons per second is the order of the minimum number of photons necessary to trigger a signal to the brain.

Example 4–3 An experimentalist wishes to remove an electron from the atom to which it is bound by sending in a single photon to collide with it. The binding energy of the electron in question is 6.05 eV. (Recall that the energy unit 1 electron volt = 1 eV = 1.6×10^{-19} J; this unit was described in Chapter 3.) What is the maximum wavelength of light necessary to remove the electron?

Solution The electron is ejected if it takes the energy of a photon of energy $E_{min} = 6.05$ eV or higher. We have seen that the energy of a photon is *inversely* proportional to its wavelength, according to $E = hf = hc/\lambda$, where we have used the wave relation $f = c/\lambda$. Inverting, we have $\lambda = hc/E$, so that there is a maximum wavelength that will allow the electron to be ejected, namely,

$$\lambda_{max} = \frac{hc}{E_{min}} = \frac{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(3.00 \times 10^{8}\,\text{m/s})}{6.05\,\text{eV}} = \frac{1.99 \times 10^{-25}\,\text{J}\cdot\text{m}}{6.05\,\text{eV}}$$

$$= \frac{1.99 \times 10^{-25}\,\text{J}\cdot\text{m}}{6.05\,\text{eV}} \frac{1\,\text{eV}}{1.6 \times 10^{-19}\,\text{J}} = 2.05 \times 10^{-7}\,\text{m}.$$

This wavelength, 205 nm, is in the ultraviolet range.

The preceding example suggests that it is useful to have an idea of the energies of photons corresponding to various wavelengths. In Table 4–1 we give a representative sampling.

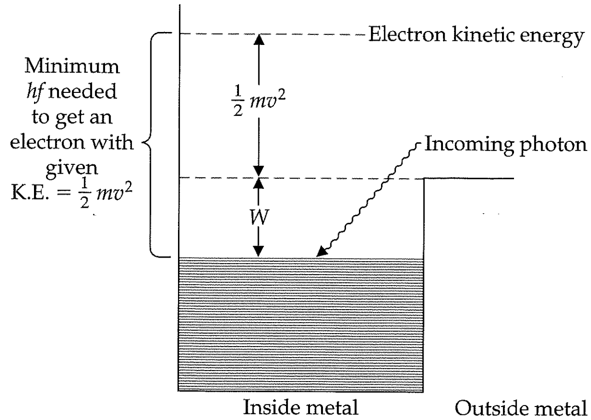
Table 4–1 Orders of Magnitudes of Single-Photon Energies Corresponding to Different Parts of the Electromagnetic Spectrum.

	Frequency (s^{-1})	Wavelength (m)	Photon Energy (J)	Photon Energy (eV)
AM radio	10^6	300	7×10^{-28}	5×10^{-9}
FM radio, TV	10^8	3	7×10^{-26}	5×10^{-7}
microwaves	10^{10}	3×10^{-2}	7×10^{-24}	5×10^{-6}
visible light	6×10^{14}	5×10^{-7}	4×10^{-19}	2.5
X rays	10^{18}	3×10^{-10}	7×10^{-16}	5×10^{3}
gamma rays from nuclear decay	10^{21}	3×10^{-13}	7×10^{-13}	5×10^{6}

At this point you may well ask, What about Maxwell's equations, and what about interference and diffraction? What happened to all the experimental evidence that light was a wave? Were these experimenters wrong? How was Einstein able to reconcile the particle nature of light with its wave nature, something that is exhibited even in the basic equation $E = hf$, whose left side apparently refers to the energy of a particle and whose right side refers to a wave property? The answers to such questions are provided by quantum mechanics, and we shall be thinking about them in chapters to come. First, however, let us discuss some results that follow from the particle nature of radiation.

4–2 The Photoelectric Effect

Metals contain a large number of free electrons (we denote their mass as m_e and their electric charge as $-e$), about one or two per atom. These electrons are called free because they are not bound to the atoms in the same way that plan-



• **Figure 4–5** The mechanism of the photoelectric effect as envisaged by Einstein.

ets are bound to the sun. However, they are not free to leave the metal: Metals do not "leak" electrons. It takes a certain amount of energy to get an electron out of a metal. You can visualize the situation as a collection of marbles rolling around in a frying pan. The marbles are free to roll around inside, but they do not have enough energy to scale the sides of the frying pan.

When a photon strikes one of these free electrons, the photon may, under the right circumstances, be absorbed by the electron, transferring enough energy to the electron to allow it to escape the metal. Electrons emitted by a metal subject to radiation are called **photoelectrons**. Following Einstein's 1905 paper, we now make the simple assumption that a single photon can be absorbed only by a single electron. A truly free isolated electron cannot absorb a photon and remain an electron, since this would violate the conservation of energy or of momentum. (See Example 3–8.) But this is not a problem here, because the struck electron can transfer momentum to the metal as a whole.

On the basis of this picture (•Fig. 4–5) we can say something about the current of photoelectrons:

- If the photon's energy is too small to boost the electron out of the metal, there will be no current at all. The energy required to do this job—analogous to that required to boost a marble the height of the sides of the frying pan—is known as the **work function** W of the metal; it varies from metal to metal and can also depend on the condition of the surface. Typical values of the work function range from 2 to 8 eV. If the frequency f of the light shining on the metal is such that $hf < W$, there will be no photoelectric current. In other words, for some metals, a weak beam of blue light produces a photocurrent, while a very intense red light produces none. If hf is larger than W, then the electrons will emerge with a speed v such that

$$\frac{1}{2}m_e v^2 = hf - W. \qquad (4\text{–}7)$$

Thus, the energy of the photoelectrons from a particular metal depends only on the frequency of the radiation, and once the threshold frequency is exceeded, the dependence of the electron's kinetic energy on the frequency of the photoelectron is linear.[3] The kinetic energy of the electron is *independent*

[3] In accordance with the particular metal involved, the electrons within have a variety of energies, and accordingly, the electrons that come out when light of a given frequency hits the material also have a variety of energies. Thus, the linear dependence on the frequency is revealed only by referring to the *maximum* photoelectron energy. For simplicity, we shall continue to refer in the text to the kinetic energy, rather than the maximum kinetic energy.

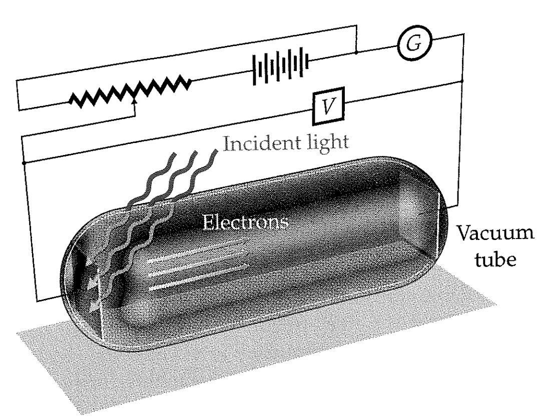
of the intensity of the radiation beam—that is, of the number of photons. This independence follows from the fact that an electron will be liberated by a single photon when the photon hits the electron. How is the kinetic energy of the photoelectron measured? This can be done by sending the photoelectron into a region in which there is a potential difference. If that potential difference V is adjusted so that no current flows, then $mv^2/2 = eV$. Once one knows the photoelectron energy, one can plot it against the frequency of the incident light. This plot should be—and is—a straight line.

Contrast this picture with the classical one, in which the energy carried by light depends on the square of the amplitude of the fields. No matter how small the frequency of the light, no matter how small the intensity, if one waits long enough, electrons will accumulate enough electromagnetic radiation to overcome the work function and escape from the metal.

- The foregoing reasoning also indicates that the *number* of photoelectrons emitted is proportional to the intensity of the radiation beam—that is, to the number of photons that shine on the metal. Again, this is not at all characteristic of the classical picture.
- There should be no time interval between the impact of the photon beam on the metal and the beginning of the emission of photoelectrons. This notion is in contrast to the classical picture, in which radiant energy arrives continuously, accumulating until there is enough to liberate an electron.

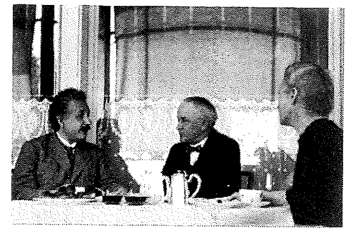
The German Heinrich Hertz, the same man who was able to verify Maxwell's classical prediction of electromagnetic waves, discovered the photoelectric effect in 1887. It was one of those accidental discoveries, made in this case while Hertz sought to improve his measurements of Maxwell's waves. Hertz's observations were of currents that were produced when light shone on metals. But not only did Hertz not know about photons; he didn't even know about the electron! Hertz's photoelectric effect attracted wide attention and further experiments, the sequence of which led eventually to J. J. Thomson's discovery of the electron in 1897. Many of the phenomena associated with the photoelectric effect were seen by Hertz. Probably, he and others were not much bothered by the difficulties of understanding the data in terms of Maxwell's classical picture of radiation without an understanding of the electron.

The final, definitive experiments on the photoelectric effect (•Fig. 4–6) were carried out in Chicago in the years 1904–1913 by Robert Millikan (•Fig. 4–7). Until these measurements, Einstein's explanation was not very widely accepted, because the experiments themselves were rather imprecise. In particular, systematic tests of the dependence of the photoelectric effect on frequency were

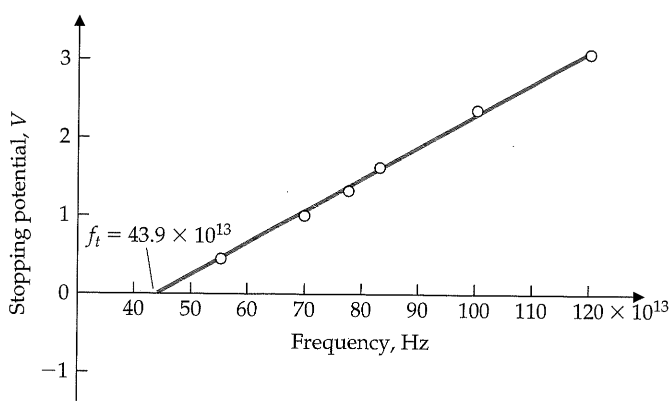


• **Figure 4–6** Robert's Millikan's experiments established the photoelectric effect in detail. The voltage V is adjusted to stop the current, so that the kinetic energy of the fastest electrons is just eV. The most important aspect of Millikan's work was his recognition of the necessity of avoiding oxidation of the surface by working in vacuum.

very difficult, and for that matter, the identification of the constant h in Eq. (4–7) with the constant introduced by Planck in connection with blackbody radiation was tenuous at best. In performing the measurements, there were problems of surface contamination. Worse, the range of light frequencies typically available made it important to use alkali metals such as sodium or potassium—ordinary metals require ultraviolet frequencies—and the alkalis are very dangerously flammable. To solve these problems, Millikan constructed what he called a "machine shop *in vacuo*." In a vacuum, he could cut shavings of these metals to expose fresh surfaces; he manipulated both the knife and the shavings by means of electromagnets. His experiments of the variation of the photoelectron energy with frequency were so precise that he could verify the linearity of this energy and measure its slope well enough to provide what was then the best measurement of Planck's constant that was available (•Fig. 4–8). Initially, Millikan did not care very much for Einstein's ideas on photons—in fact, he even commented that he thought, erroneously, that Einstein himself had abandoned those ideas—but despite himself, he succeeded admirably in verifying the ideas.



• Figure 4–7 Robert Millikan, in company with Albert Einstein and Marie Curie.



• Figure 4–8 Millikan's data on the photoelectric effect. Note the linearity in the plot of the energy of the photoelectron versus the frequency of the incident light.

Example 4–4 An experiment shows that when electromagnetic radiation of wavelength 270 nm falls on an aluminum surface, photoelectrons are emitted. The most energetic of these are stopped by a potential difference of 0.406 volts. Use this information to calculate the work function of aluminum in electron volts.

Solution The kinetic energy of the most energetic photoelectrons is given by the electron charge times the potential that stops the photoelectrons:

$$K = eV = (1.6 \times 10^{-19}\,\text{C})(0.406\,\text{V}) = 0.65 \times 10^{-19}\,\text{J}.$$

The photon energy is

$$E = hf = \frac{hc}{\lambda} = (6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(3.00 \times 10^{8}\,\text{m/s})/(270 \times 10^{-9}\,\text{m}) = 7.37 \times 10^{-19}\,\text{J}.$$

The difference is the work function:

$$W = E - K = 6.72 \times 10^{-19}\,\text{J} = (6.72 \times 10^{-19}\,\text{J})/(1.6 \times 10^{-19}\,\text{J/eV}) = 4.2\,\text{eV}.$$

The photoelectric effect has some important applications, inasmuch as it lies behind many devices that react to light signals. The camera exposure meter and light-activated keys for automobiles, distant television controls, or garage-door openers are some examples that come to mind. The photomultiplier described in •Fig. 4–4 depends on the photoelectric effect to initiate the cascade of electrons within it.