5.2 De Broglie Waves


AIP/Niels Bohr Library, W. F. Meggers Collection.

After serving in World War I, **Prince Louis V. de Broglie** (1892–1987) resumed his studies toward a doctoral degree at the University of Paris in 1924, where he reported his concept of matter waves as part of his doctoral dissertation. De Broglie spent his life in France where he enjoyed much success as an author and teacher.

By 1920 it was established that x rays were electromagnetic radiation that exhibited wave properties. X-ray crystallography and its usefulness in studying the crystalline structure of atoms and molecules was being established. However, a detailed understanding of the atom was still lacking. Many physicists believed that a new, more general theory was needed to replace the rudimentary Bohr model of the atom. An essential step in this development was made by a young French graduate student, Prince Louis V. de Broglie, who began studying the problems of the Bohr model in 1920.

De Broglie was well versed in the work of Planck, Einstein, and Bohr. He was aware of the duality of nature expressed by Einstein in which matter and energy were not independent but were in fact interchangeable. De Broglie was particularly struck by the fact that photons (electromagnetic radiation) had both wave (x-ray crystallography) and corpuscular (photoelectric effect) properties. The concept of waves is needed to understand interference and diffraction (Section 5.1), but localized corpuscles are needed to explain phenomena like the photoelectric effect (Section 3.6) and Compton scattering (Section 3.8). If electromagnetic radiation must have *both wave and particle properties*, then why should material particles not have both wave and particle properties as well? According to de Broglie, the symmetry of nature encourages such an idea, and no laws of physics prohibit it.

When de Broglie presented his new hypothesis in a doctoral thesis to the University of Paris in 1924, it aroused considerable interest. De Broglie used Einstein's special theory of relativity together with Planck's quantum theory to establish the wave properties of particles. His fundamental relationship is the prediction

De Broglie wavelength of a particle

$$\lambda = \frac{h}{p} \tag{5.2}$$

That is, the wavelength to be associated with a particle is given by Planck's constant divided by the particle's momentum.

De Broglie was guided by the concepts of phase and group velocities of waves (see Section 5.4) to arrive at Equation (5.2). Recall that for a photon $E = pc$, and $E = hf$, so that

$$hf = pc = p\lambda f$$

$$h = p\lambda$$

and

$$\lambda = \frac{h}{p} \tag{5.3}$$

Matter waves

De Broglie extended this relation for photons to all particles. Particle waves were called **matter waves** by de Broglie, and the wavelength expressed in Equation (5.2) is now called the **de Broglie wavelength** of a particle.

EXAMPLE 5.2

Calculate the de Broglie wavelength of (a) a tennis ball of mass 57 g traveling 25 m/s (about 56 mph) and (b) an electron with kinetic energy 50 eV.

Strategy The calculation for both of these wavelengths is a straightforward application of Equation (5.2).

Solution (a) For the tennis ball, $m = 0.057$ kg, so

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\,\text{J} \cdot \text{s}}{(0.057\ \text{kg})(25\ \text{m/s})} = 4.7 \times 10^{-34}\ \text{m}$$

(b) For the electron, it is more convenient to use eV units, so we rewrite the wavelength λ as

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{hc}{\sqrt{2(mc^2)K}}$$

$$\lambda = \frac{1240\ \text{eV} \cdot \text{nm}}{\sqrt{(2)(0.511 \times 10^6\ \text{eV})(50\ \text{eV})}} = 0.17\ \text{nm}$$

Note that because the kinetic energy of the electron is so small, we have used a nonrelativistic calculation. Calculations in modern physics are normally done using eV units, both because it is easier and also because eV values are more appropriate for atoms and nuclei (MeV, GeV) than are joules. The values of hc and some masses can be found inside the front cover.

How can we show whether such objects as the tennis ball or the electron in the previous example exhibit wavelike properties? The best way is to pass the objects through a slit having a width of the same dimension as the object's wavelength. We expect it to be virtually impossible to demonstrate interference or diffraction for the tennis ball, because we cannot find a slit as narrow as 10^{-34} m. It is unlikely we will ever be able to demonstrate the wave properties of the tennis ball. But the de Broglie wavelength of the 50-eV electron, about 0.2 nm, is large enough that we should be able to demonstrate its wave properties. Because of their small mass, electrons can have a small momentum and in turn a large wavelength ($\lambda = h/p$). Electrons offer our best chance of observing effects due to matter waves.

Bohr's Quantization Condition

One of Bohr's assumptions concerning his hydrogen atom model was that the angular momentum of the electron-nucleus system in a stationary state is an integral multiple of $h/2\pi$. Let's now see if we can predict this result using de Broglie's result. Represent the electron as a standing wave in an orbit around the proton. The condition for a standing wave in this configuration is that the entire length of the standing wave must just fit around the orbit's circumference. We show an example of this in Figure 5.8. In order for it to be a correct standing wave, we must have

$$n\lambda = 2\pi r$$

where r is the radius of the orbit. Now we use the de Broglie relation for the wavelength and obtain

$$2\pi r = n\lambda = n\frac{h}{p}$$

The angular momentum of the electron in this orbit is $L = rp$, so we have, using the above relation,

$$L = rp = \frac{nh}{2\pi} = n\hbar$$

We have arrived at Bohr's quantization assumption by simply applying de Broglie's wavelength for an electron in a standing wave. This result seemed to justify Bohr's assumption. De Broglie's wavelength theory for particles was a crucial step toward the new quantum theory, but experimental proof was lacking. As we will see in the next section, this was soon to come.

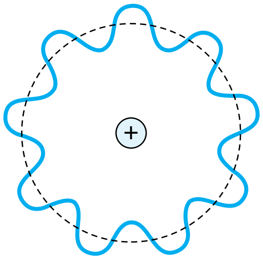

Figure 5.8 A schematic diagram of standing waves in an electron orbit around a nucleus. An integral number of wavelengths fits in the orbit. Note that the electron does not "wiggle" around the nucleus. The displacement from the dashed line represents its wave amplitude.

Cavendish Laboratory

Before the 1870s most of our scientific knowledge resulted from the research of people working in their own private laboratories. William Thomson (who would later become Lord Kelvin) established a laboratory at the University of Glasgow in the 1840s, and in the 1860s efforts began at both Oxford and Cambridge to build physical laboratories. In 1871 James Clerk Maxwell was called from his Scottish home to become the first Cavendish Professor at Cambridge University. Maxwell began planning and supervising the construction of the laboratory on Free School Lane in central Cambridge with an unexpected fervor while he gave regular lectures to students. The most important work of the day was to demonstrate the existence of Maxwell's electromagnetic waves, but they were "scooped" by Heinrich Hertz in Germany. Maxwell's successor was Lord Rayleigh, who published 50 papers during his five years at Cavendish before returning to his estate farm where he made most of his discoveries (including the noble gases) at his private laboratory.

The appointment of the young J. J. Thomson at age 28 as Cavendish Professor in 1884 was the beginning of a long and fruitful era in atomic physics. The discovery of the electron in 1897, the arrival of the young Ernest Rutherford from New Zealand as a student, and the early work of C. T. R. Wilson that led to the development of the cloud chamber all helped the Cavendish Laboratory expand, prosper, and grow in stature under Thomson's leadership. Thomson's 35-year leadership was remarkable in many ways, particularly in the manner he stepped down in 1919 upon the opportunity of attracting Rutherford back to Cavendish to be the next Professor.

During Rutherford's 19-year reign, the Cavendish became the most renowned center of science in the world. It attracted the best students, researchers, and visitors from all over the world. Rutherford was a team leader, and he surrounded himself with a collection of young physicists whom he called "his boys." By the end of the Rutherford era in 1937, the laboratory was moving into new directions with particle accelerators and cryogenic labs.

World War II would change the face of the Cavendish forever. Physicists spread out to perform wartime research, particularly on the development of the atomic bomb and radar, both of which played large roles in the allied victory. William Lawrence Bragg returned to Cambridge as Cavendish Professor to succeed Rutherford in 1937, and the field of x-ray crystallography flourished. The Cavendish scientists have had an uncanny ability to choose productive research areas. It has been said that the fields of molecular biology and radio astronomy started at the Cavendish in the late 1940s, and Bragg must be given credit for the foresight to support these fledgling subjects in the face of "Big Science" in the United States. Bragg's tenure as Cavendish Professor ended in 1953 just when Watson and Crick succeeded in discovering the DNA structure. Bragg also supported J. A. Ratcliffe and Martin Ryle, who had worked on radar at the Cavendish during the war, to construct the first radio telescope. This effort led to the discovery of quasars and pulsars.

When Sir Nevill Mott succeeded Bragg as Cavendish Professor in 1954, the lab made a turn toward solid state physics. Mott had worked on collision theory and nuclear problems in the 1930s but eventually turned to theoretical investigations of electronic systems. Brian Josephson did his pioneering theoretical work (see Chapter 10) on the supercurrent through a tunnel barrier while a student, graduating in 1964 with his Ph.D. In 1974 the Cavendish moved to a new site in West Cambridge. Condensed matter physics now accounts for the greater part of research at the Cavendish, but the groups in radio astronomy and high-energy physics are still important. The Cavendish Laboratory has set a standard that other laboratories can only hope to emulate.

We end with a list of Nobel Prizes awarded to those who did their most important work at the Cavendish Laboratory. The asterisks (for example, Rutherford and Rayleigh) indicate Nobel Prizes awarded primarily for work done elsewhere to people who are still widely associated with the Cavendish Laboratory.

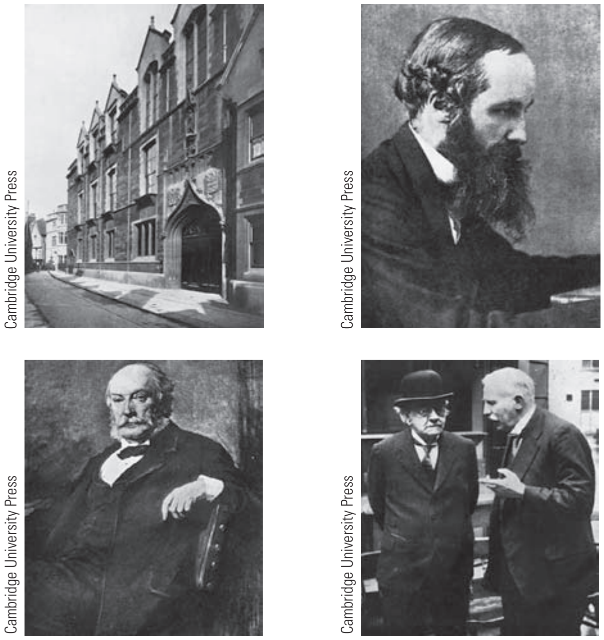

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Figure A Upper left, the old Cavendish Laboratory on Free School Lane in Cambridge. The original building is to the left of the gate. The first four Cavendish professors: James Clerk Maxwell, upper right; Lord Rayleigh, bottom left; and Sir J. J. Thomson (left) and Lord Rutherford, bottom right.

Cavendish Laboratory Nobel Prizes

1904	Physics	Lord Rayleigh*	Density of gases, discovery of argon
1906	Physics	Sir J. J. Thomson	Investigations of electricity in gases
1908	Chemistry	Lord Rutherford*	Element disintegration
1915	Physics	Sir William Lawrence Bragg	X-ray analysis of crystals
1917	Physics	Charles G. Barkla	Secondary x rays
1922	Chemistry	Francis W. Aston	Isotopes discovery
1927	Physics	Charles T. R. Wilson	Cloud chamber
1928	Physics	Sir Owen W. Richardson	Thermionic emission
1935	Physics	Sir James Chadwick	Neutron discovery
1937	Physics	Sir George P. Thomson	Electron diffraction
1947	Physics	Sir Edward V. Appleton*	Upper atmosphere investigations
1948	Physics	Lord Patrick M. S. Blackett	Discoveries in nuclear physics
1951	Physics	Sir John D. Cockcroft and Ernest T. S. Walton	Nuclear transmutation
1962	Physiology or Medicine	Francis H. C. Crick and James D. Watson	DNA discoveries
1962	Chemistry	Max Perutz and Sir John Kendrew	Structures of globular proteins
1973	Physics	Brian D. Josephson	Supercurrent in tunnel barriers
1974	Physics	Sir Martin Ryle and Antony Hewish	Radio astrophysics, pulsars
1977	Physics	Sir Nevill F. Mott	Magnetic and disordered systems
1978	Physics	P. L. Kapitsa*	Low-temperature physics
1982	Chemistry	Sir Aaron Klug	Nucleic acid–protein complexes

AIP/Emilio Segrè Visual Archives.

Clinton J. Davisson (1881–1958) is shown here in 1928 (right) looking at the electronic diffraction tube held by **Lester H. Germer** (1896–1971). Davisson received his undergraduate degree at the University of Chicago and his doctorate at Princeton. They performed their work at Bell Telephone Laboratory located in New York City. Davisson received the Nobel Prize in Physics in 1937.

5.3 Electron Scattering

In 1925 a laboratory accident led to experimental proof for de Broglie's wavelength hypothesis. C. Davisson and L. H. Germer of Bell Telephone Laboratories (now part of Alcatel-Lucent) were investigating the properties of metallic surfaces by scattering electrons from various materials when a liquid air bottle exploded near their apparatus. Because the nickel target they were currently using was at a high temperature when the accident occurred, the subsequent breakage of their vacuum system caused significant oxidation of the nickel. The target had been specially prepared and was rather expensive, so they tried to repair it by, among other procedures, prolonged heating at various high temperatures in hydrogen and under vacuum to deoxidize it.

A simple diagram of the Davisson-Germer apparatus is shown in Figure 5.9. Upon putting the refurbished target back in place and continuing the experiments, Davisson and Germer found a striking change in the way electrons were scattering from the nickel surface. They had previously seen a smooth variation of intensity with scattering angle, but the new data showed large numbers of scattered electrons for certain energies at a given scattering angle. Davisson and Germer were so puzzled by their new data that after a few days, they cut open the tube to examine the nickel target. They found that the high temperature had modified the polycrystalline structure of the nickel. The many small crystals of the original target had been changed into a few large crystals as a result of the heat treatment. Davisson surmised it was this new crystal structure of nickel—the arrangement of atoms in the crystals, not the structure of the atoms—that had caused the new intensity distributions. Some 1928 experimental results of Davisson and Germer for 54-eV electrons scattered from nickel are shown in Figure 5.10. The scattered peak occurs for $\phi = 50°$.

The electrons were apparently being diffracted much like x rays, and Davisson, being aware of de Broglie's results, found that the Bragg law applied to their data as well. Davisson and Germer were able to vary the scattering angles for a given wavelength and vary the wavelength (by changing the electron accelerating voltage and thus the momentum) for a given angle.

The relationship between the incident electron beam and the nickel crystal scattering planes is shown in Figure 5.11. In the Bragg law, 2θ is the angle between the incident and exit beams. Therefore, $\phi = \pi - 2\theta = 2\alpha$. Because $\sin\theta = \cos(\phi/2) = \cos\alpha$, we have for the Bragg condition, $n\lambda = 2d\cos\alpha$.

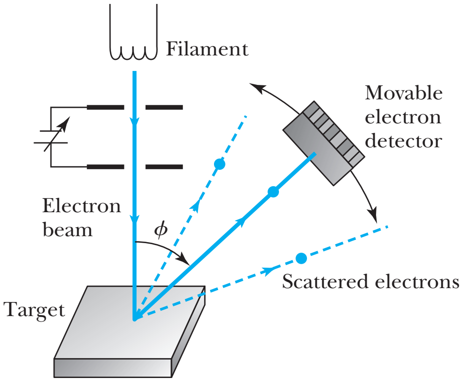


Figure 5.9 Schematic diagram of Davisson-Germer experiment. Electrons are produced by the hot filament, accelerated, and focused onto the target. Electrons are scattered at an angle ϕ into a detector, which is movable. The distribution of electrons is measured as a function of ϕ. The entire apparatus is located in a vacuum.

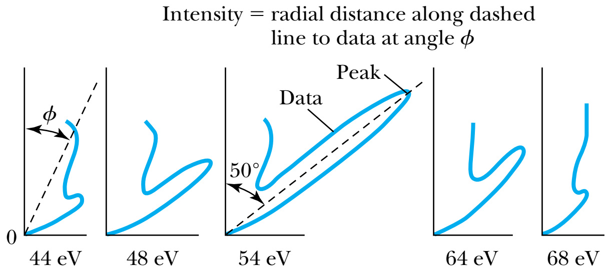


Figure 5.10 Davisson and Germer data for scattering of electrons from Ni. The peak $\phi = 50°$ builds dramatically as the energy of the electron nears 54 eV. *From C. J. Davisson,* Franklin Institute Journal ***205****, 597–623 (1928).*

However, d is the lattice plane spacing and is related to the interatomic distance D by $d = D \sin \alpha$ so that

$$n\lambda = 2d \sin \theta = 2d \cos \alpha = 2D \sin \alpha \cos \alpha$$

$$n\lambda = D \sin 2\alpha = D \sin \phi \tag{5.4}$$

or

$$\lambda = \frac{D \sin \phi}{n} \tag{5.5}$$

For nickel the interatomic distance is $D = 0.215$ nm. If the peak found by Davisson and Germer at 50° was $n = 1$, then the electron wavelength should be

$$\lambda = (0.215 \text{ nm})(\sin 50°) = 0.165 \text{ nm}$$

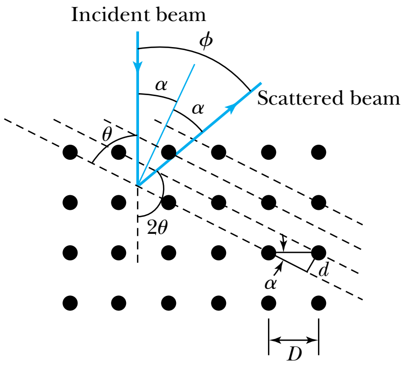


Figure 5.11 The scattering of electrons by lattice planes in a crystal. This figure is useful to compare the scattering relations $n\lambda = 2d \sin \theta$ and $n\lambda = D \sin \phi$ where θ and ϕ are the angles shown, D = interatomic spacing, and d = lattice plane spacing.

EXAMPLE 5.3

Determine the de Broglie wavelength for a 54-eV electron used by Davisson and Germer.

Strategy We shall use the de Broglie wavelength Equation (5.2) to determine the wavelength λ. We need to find the momentum of a 54-eV electron, but because the energy is so low, we can do a nonrelativistic calculation. We shall do a general calculation for the wavelength of any electron accelerated by a voltage of V_0.

Solution We write the kinetic energy K.E. in terms of the final momentum of the electron and the voltage V_0 across which the electron is accelerated.

$$\frac{p^2}{2m} = \text{K.E.} = eV_0 \tag{5.6}$$

We find the momentum from this equation to be $p = \sqrt{(2m)(eV_0)}$. The de Broglie wavelength from Equation (5.2) is now

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{\sqrt{(2mc^2)(eV_0)}}$$

$$= \frac{1240\ \text{eV} \cdot \text{nm}}{\sqrt{(2)(0.511 \times 10^6\ \text{eV})(eV_0)}}$$

$$\lambda = \frac{1.226\ \text{nm} \cdot \text{V}^{1/2}}{\sqrt{V_0}} \tag{5.7}$$

where the constants h, c, and m have been evaluated and V_0 is the voltage. For $V_0 = 54$ V, the wavelength is

$$\lambda = \frac{1.226\ \text{nm} \cdot \text{V}^{1/2}}{\sqrt{54\ \text{V}}} = 0.167\ \text{nm}$$

We note that the value of the de Broglie wavelength 0.167 nm found in the previous example is in good agreement with that found experimentally (0.165 nm) by Davisson and Germer for the peak at 50°. This is an important result and shows that electrons have wavelike properties.

Shortly after Davisson and Germer reported their experiment, George P. Thomson (1892–1975), son of J. J. Thomson, reported seeing the effects of electron diffraction in transmission experiments. The first target was celluloid, and soon after that gold, aluminum, and platinum were used. The randomly oriented polycrystalline sample of beryllium produces rings (see Figure 5.12b). Davisson and Thomson received the Nobel Prize in 1937 for their investigations, which clearly showed that particles exhibited wave properties. In the next few years hydrogen and helium atoms were also shown to exhibit wave diffraction. An important modern measurement technique uses diffraction of neutrons to study the crystal and molecular structure of biologically important substances. All these experiments are consistent with the de Broglie hypothesis for the wavelength of a particle with mass.

EXAMPLE 5.4

In introductory physics, we learned that a particle (ideal gas) in thermal equilibrium with its surroundings has a kinetic energy of $3kT/2$. Calculate the de Broglie wavelength for (a) a neutron at room temperature (300 K) and (b) a "cold" neutron at 77 K (liquid nitrogen).

Strategy In both of these cases we will use Equation (5.2) to find the de Broglie wavelength. First, we will need to determine the momentum, and we note in both cases the energies of the particles will be so low that we can perform a nonrelativistic calculation. Neutrons have a rest energy of almost 1000 MeV, and their kinetic energies at these temperatures will be quite low (0.026 eV at 300 K).

Solution We begin by finding the de Broglie wavelength of the neutron in terms of the temperature.

$$\frac{p^2}{2m} = \text{K.E.} = \frac{3}{2}kT \tag{5.8}$$

$$p = \sqrt{3mkT}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}} = \frac{hc}{\sqrt{3(mc^2)kT}}$$

$$= \frac{1}{T^{1/2}} \frac{1240\ \text{eV} \cdot \text{nm}}{\sqrt{3(938 \times 10^6\ \text{eV})(8.62 \times 10^{-5}\ \text{eV/K})}}$$

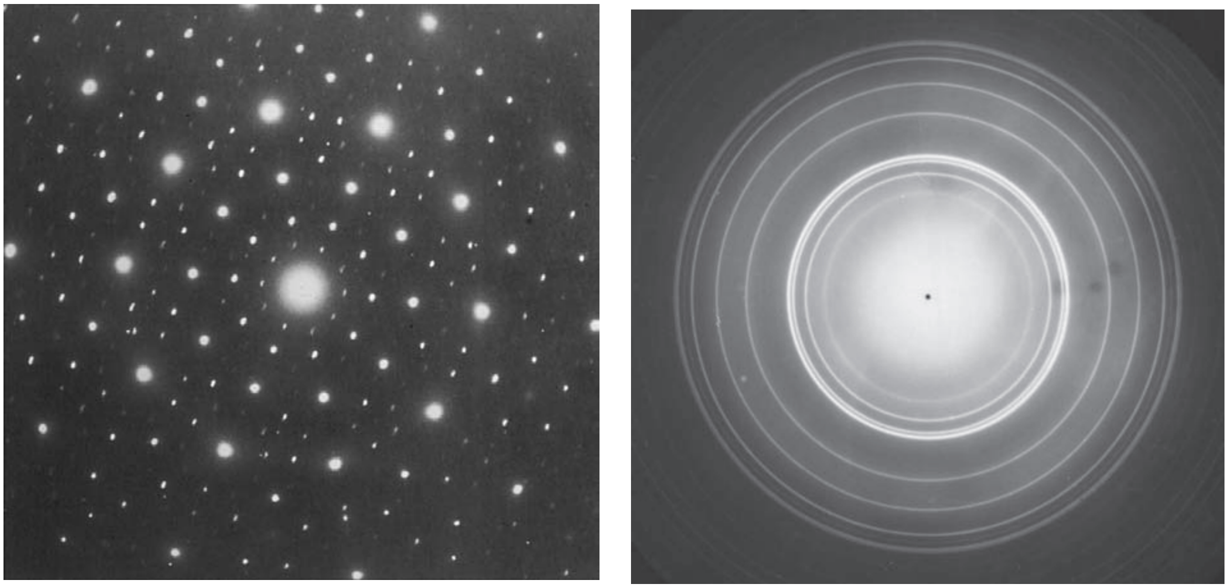
It again has been convenient to use eV units.

$$\lambda = \frac{2.52}{T^{1/2}}\ \text{nm} \cdot \text{K}^{1/2}$$

$$\lambda(300\ \text{K}) = \frac{2.52\ \text{nm} \cdot \text{K}^{1/2}}{\sqrt{300\ \text{K}}} = 0.145\ \text{nm} \tag{5.9}$$

$$\lambda(77\ \text{K}) = \frac{2.52\ \text{nm} \cdot \text{K}^{1/2}}{\sqrt{77\ \text{K}}} = 0.287\ \text{nm}$$

These wavelengths are thus suitable for diffraction by crystals. "Supercold" neutrons, used to produce even larger wavelengths, are useful because extraneous electric and magnetic fields do not affect neutrons nearly as much as electrons.



(a)

(b)

Courtesy of David Follstaedt, Sandia National Laboratory

Omikron/Photo Researchers, Inc.

Figure 5.12 Examples of transmission electron diffraction photographs. (a) Produced by scattering 120-keV electrons on the quasicrystal $Al_{80}Mn_{20}$. (b) Electron diffraction pattern on beryllium. Notice that the dots in (a) indicate that the sample was a crystal, whereas the rings in (b) indicate that a randomly oriented sample (or powder) was used.

5.4 Wave Motion

Because particles exhibit wave behavior, as shown in the last section for electron diffraction, it must be possible to formulate a wave description of particle motion. This is an essential step in our progress toward understanding the behavior of matter—the quantum theory of physics. Our development of quantum theory will be based heavily on waves, so we now digress briefly to review the physics of wave motion, which we shall soon apply to particles.

In introductory physics, we study waves of several kinds, including sound waves and electromagnetic waves (including light). The simplest form of wave has a sinusoidal form; at a fixed time (say, $t = 0$) its spatial variation looks like

$$\Psi(x, t)\big|_{t=0} = A \sin\left(\frac{2\pi}{\lambda}x\right) \tag{5.10}$$

as shown in Figure 5.13 (p. 176). The function $\Psi(x, t)$ represents the *instantaneous amplitude* or **displacement** of the wave as a function of position x and time t. In the case of a traveling wave moving down a string, Ψ is the displacement of the string from equilibrium; and in the case of electromagnetic radiation, Ψ is the magnitude of the electric field $\vec{E}$ or magnetic field $\vec{B}$. The maximum displacement A is normally called the **amplitude**, but a better term for a harmonic wave such as we are considering may be **harmonic amplitude**.

As time increases, the position of the wave will change, so the general expression for the wave is

$$\Psi(x, t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right] \tag{5.11}$$

Wave form