> although I have several times observed drops which in my judgement lasted considerably longer than this. The drops which it was found possible to balance by an electric field always carried multiple charges, and the difficulty experienced in balancing such drops was less than had been anticipated.[6]

The discovery that he could see individual droplets and that droplets suspended in a vertical electric field sometimes suddenly moved upward or downward, evidently because they had picked up a positive or negative ion, led to the possibility of observing the charge of a single ion. In 1909, Millikan began a series of experiments which not only showed that charges occurred in integer multiples of an elementary unit e, but measured the value of e to about 1 part in 1000. To eliminate evaporation, he used oil drops sprayed into dry air between the plates of a capacitor. These drops were already charged by the spraying process, i.e., friction in the spray nozzle, and during the course of observation they picked up or lost additional charges. By switching the field between the plates, a drop could be moved up or down and observed for several hours. When the charge on a drop changed, the velocity of the drop with the field "on" changed. Assuming only that the terminal velocity of the drop was proportional to the force acting on it (this assumption was carefully checked experimentally), Millikan's experiment gave conclusive evidence that charges always occur in multiples of a fundamental unit e, whose value he determined to be 1.601×10^{-19} C. The currently accepted value is, to three decimal places, 1.602×10^{-19} C. The expanded discussion of Millikan's experiment on the home page includes the value to eight places.

More

Millikan's Oil-Drop Experiment,[7] one of the few truly crucial experiments in physics, is also remarkable for its simple directness and its excellent precision. The discussion of Millikan's experiment on our home page includes a portion of the data on drop number 6, one of several thousand oil drops he used in determining the value of the electron's charge. See also Equations 3-10 through 3-18 and Figures 3-4 and 3-5 on the home page: www.whfreeman.com/modphysics4e

3-2 Blackbody Radiation

The first clue to the quantum nature of radiation came from the study of thermal radiation emitted by opaque bodies. When radiation falls on an opaque body, part of it is reflected and the rest absorbed. Light-colored bodies reflect most of the visible radiation incident on them, whereas dark bodies absorb most of it. The absorption part of the process can be described briefly as follows. The radiation absorbed by the body increases the kinetic energy of the constituent atoms which oscillate about their equilibrium positions. Recalling that the average translational kinetic energy of the atoms determines the temperature of the body, the absorbed energy causes the temperature to rise. However, the atoms contain charges (the electrons) and they are accelerated by the oscillations. Consequently, as required by electromagnetic theory, the atoms emit electromagnetic radiation which reduces the kinetic energy of the oscillations and tends to reduce the temperature. When the rate of absorption equals that of emission, the temperature is constant and we say that the body is in thermal equilibrium with its surroundings. A good absorber of radiation is therefore also a good emitter.

The electromagnetic radiation emitted under these circumstances is called *thermal radiation.* At ordinary temperatures (below about 600°C) the thermal radiation emitted by a body is not visible; most of the energy is concentrated in wavelengths much longer than those of visible light. As a body is heated, the quantity of thermal radiation emitted increases, and the energy radiated extends to shorter and shorter wavelengths. At about 600–700°C there is enough energy in the visible spectrum so that the body glows and becomes a dull red, and at higher temperatures it becomes bright-red or even "white-hot."

A body that absorbs *all* radiation incident on it is called an *ideal blackbody.* In 1879 Josef Stefan found an empirical relation between the power per unit area radiated by a blackbody and the temperature:

$$R = \sigma T^4 \qquad \textbf{3-19}$$

where R is the power radiated per unit area, T is the absolute temperature, and $\sigma = 5.6703 \times 10^{-8}\ \mathrm{W/m^2K^4}$ is a constant called Stefan's constant. This result was also derived on the basis of classical thermodynamics by Ludwig Boltzmann about five years later, and Equation 3-19 is now called the Stefan-Boltzmann law. Note that the power per unit area radiated by a blackbody depends only on the temperature, and not on any other characteristic of the object, such as its color or the material of which it is composed. Note, too, that R tells us the *rate* at which energy is emitted by the object. For example, doubling the absolute temperature of an object increases the energy flow out of the object by a factor of $2^4 = 16$. An object at room temperature (300 K) will double the rate at which it radiates energy as a result of a temperature increase of only 57°C. Thus, the Stefan-Boltzmann law has an enormous effect on the establishment of thermal equilibrium in physical systems.

Objects that are not blackbodies radiate energy per unit area at a rate less than that of a blackbody at the same temperature. The rate does depend on properties in addition to the temperature, such as color and composition of the surface. The effects of those dependencies are combined into a factor called the *emissivity* ϵ which multiplies the right side of Equation 3-19. The values of ϵ, which is itself temperature dependent, are always less than unity.

Like the total radiated power R, the *spectral distribution* of the radiation emitted by a blackbody is found empirically to depend *only* on the absolute temperature T. The spectral distribution is determined experimentally as illustrated schematically in Figure 3-6. Let $R(\lambda)d\lambda$ be the power emitted per unit area with wavelength between

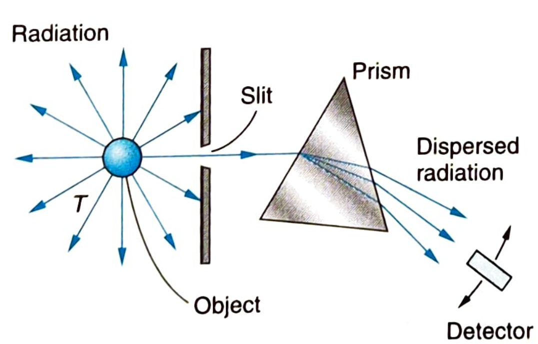

Fig. 3-6 Radiation emitted by the object at temperature T that passes through the slit is dispersed according to its wavelength. The prism shown would be an appropriate device for that part of the emitted radiation in the visible region. In other spectral regions other types of devices or wavelength-sensitive detectors would be used.

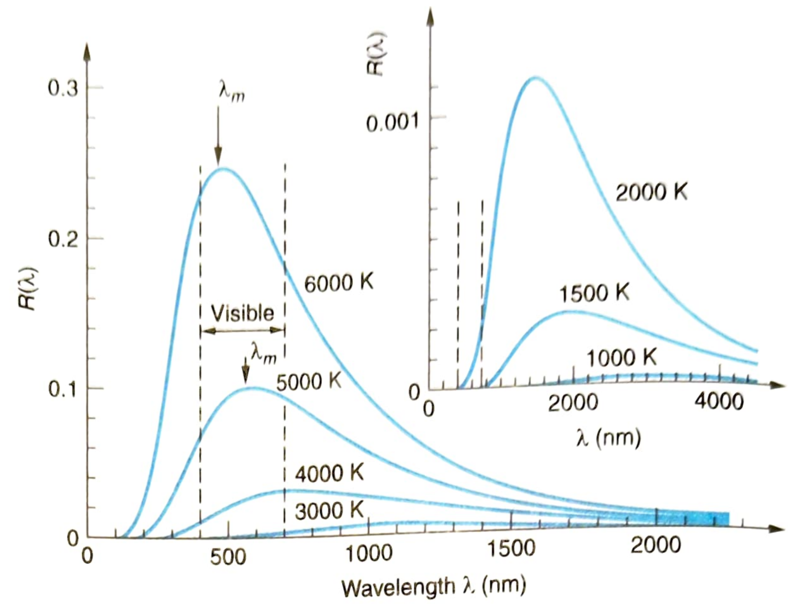

Fig. 3-7 Spectral distribution function $R(\lambda)$ measured at different temperatures. The $R(\lambda)$ axis is in arbitrary units for comparison only. Notice the range in λ of the visible spectrum. The sun emits radiation very close to that of a blackbody at 5800 K. λ_m is indicated for the 5000-K and 6000-K curves.

λ and $\lambda + d\lambda$. Figure 3-7 shows the measured spectral distribution function $R(\lambda)$ versus λ for several values of T ranging from 1000 K to 6000 K.

The $R(\lambda)$ versus λ curves in Figure 3-7 are quite remarkable in several respects. One is that the wavelength at which the distribution is maximum varies inversely with the temperature:

$$\lambda_m \propto \frac{1}{T}$$

or

$$\lambda_m T = \text{constant} = 2.898 \times 10^{-3}\ \text{mK} \qquad \textbf{3-20}$$

This result is known as Wien's displacement law. It was obtained by Wilhelm Wien in 1893. Examples 3-2 and 3-3 illustrate its application.

EXAMPLE 3-2 **How Big Is a Star?** Measurement of the wavelength at which the spectral distribution $R(\lambda)$ from a certain star is maximum indicates that the star's surface temperature is 3000 K. If that star is also found to radiate 100 times the power radiated by the sun $P_\odot$, how big is the star? (The symbol $\odot$ = sun.) The sun's surface temperature is found to be 5800 K.

Solution

Assuming the sun and the star both radiate as blackbodies (astronomers nearly always make this assumption, based on, among other things, the fact that the solar spectrum is nearly that of a perfect blackbody), their surface temperatures have been determined from Equation 3-20 to be 5800 K and 3000 K, respectively. Measurement also indicates that $P_{\text{star}} = 100\ P_\odot$. Thus, from Equation 3-19 we have that

$$R_{star} = \frac{P_{star}}{(\text{area})_{star}} = \frac{100P_{\odot}}{4\pi r_{star}^2} = \sigma T_{star}^4$$

and

$$R_{\odot} = \frac{P_{\odot}}{(\text{area})_{\odot}} = \frac{P_{\odot}}{4\pi r_{\odot}^2} = \sigma T_{\odot}^4$$

Thus, we have

$$r_{star}^2 = 100 r_{\odot}^2 \left(\frac{T_{\odot}}{T_{star}}\right)^4$$

$$r_{star} = 10 r_{\odot} \left(\frac{T_{\odot}}{T_{star}}\right)^2 = 10\left(\frac{5800}{3000}\right)^2 r_{\odot}$$

$$r_{star} = 37.4 r_{\odot}$$

Since $r_{\odot} = 6.96 \times 10^8$ m, this star has a radius of about 2.6×10^{10} m, or about half of the radius of the orbit of Mercury.

Rayleigh-Jeans Equation

The calculation of the distribution function $R(\lambda)$ involves the calculation of the energy density of electromagnetic waves in a cavity. Materials such as black velvet or lampblack come close to being ideal blackbodies, but the best practical realization of an ideal blackbody is a small hole leading into a cavity (such as a keyhole in a closet door; see Figure 3-8). Radiation incident on the hole has little chance of being reflected back out of the hole before it is absorbed by the walls of the cavity. The power radiated *out* of the hole is proportional to the total energy density U (energy per unit volume) of the radiation in the cavity. The proportionality constant can be shown to be $c/4$, where c is the speed of light:[9]

$$R = \frac{1}{4}cU \qquad \textbf{3-21}$$

Similarly, the spectral distribution of the power emitted from the hole is proportional to the spectral distribution of the energy density in the cavity. If $u(\lambda)d\lambda$ is the fraction

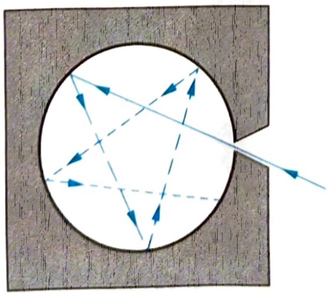

Fig. 3-8 A small hole in the wall of a cavity approximating an ideal blackbody. Radiation entering the hole has little chance of leaving before it is completely absorbed within the cavity.

of the energy per unit volume in the cavity in the range $d\lambda$, then $u(\lambda)$ and $R(\lambda)$ are related by

$$R(\lambda) = \tfrac{1}{4}cu(\lambda) \qquad \textbf{3-22}$$

The energy density distribution function $u(\lambda)$ can be calculated from classical physics in a straightforward way. The method involves finding the number of modes of oscillation of the electromagnetic field in the cavity with wavelengths in the interval $d\lambda$ and multiplying by the average energy per mode. We shall not go into the details of the calculation here. The result is that the number of modes of oscillation per unit volume, $n(\lambda)$, is independent of the shape of the cavity and is given by

$$n(\lambda) = 8\pi\lambda^{-4} \qquad \textbf{3-23}$$

According to classical kinetic theory, the average energy per mode of oscillation is kT, the same as for a one-dimensional harmonic oscillator, where k is the Boltzmann constant. Classical theory thus predicts for the energy density spectral distribution function

$$u(\lambda) = kTn(\lambda) = 8\pi kT\lambda^{-4} \qquad \textbf{3-24}$$

This prediction, initially derived by Lord Rayleigh,[10] is called the *Rayleigh-Jeans law,* and is illustrated in Figure 3-9.

At very long wavelengths the Rayleigh-Jeans law agrees with the experimentally determined spectral distribution, but at short wavelengths this law predicts that $u(\lambda)$ becomes large, approaching infinity as $\lambda \rightarrow 0$, whereas experiment shows (see Figures 3-7 and 3-9) that the distribution actually approaches zero as $\lambda \rightarrow 0$. This enormous disagreement between the experimental measurement of $u(\lambda)$ and the prediction of the fundamental laws of classical physics at short wavelengths was called the *ultraviolet catastrophe.* The word *catastrophe* was not used lightly: Equation 3-24 implies that

$$\int_0^\infty u(\lambda)d\lambda \longrightarrow \infty \qquad \textbf{3-25}$$

i.e., every object would have an infinite energy density.

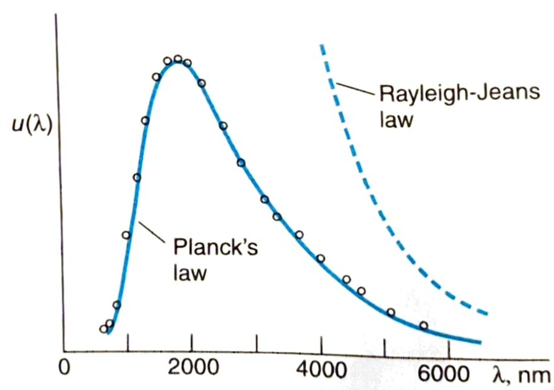

Fig. 3-9 Comparison of Planck's law and the Rayleigh-Jeans law with experimental data at $T = 1600$ K obtained by W. W. Coblentz in about 1915. The $u(\lambda)$ axis is linear. [*Adapted from F. K. Richtmyer, E. H. Kennnard, and J. N. Cooper,* Introduction to Modern Physics, *6th ed. (New York: McGraw-Hill Book Company, 1969), by permission.*]

Planck's Law

In 1900 the German physicist Max Planck[11] announced that by making somewhat strange assumptions, he could derive a function $u(\lambda)$ that agreed with the experimental data. He first found an empirical function that fit the data, and then searched for a way to modify the usual calculation so as to predict his empirical formula. We can see the type of modification needed if we note that, for any cavity, the shorter the wavelength, the more standing waves (modes) will be possible. As $\lambda \rightarrow 0$ the number of modes of oscillation approaches infinity, as evidenced in Equation 3-23. In order for the energy density distribution function $u(\lambda)$ to approach zero, we expect the average energy per mode to depend on the wavelength λ and approach zero as λ approaches zero, rather than be equal to the value kT predicted by classical theory.

Parenthetically, we should observe that those working on the ultraviolet catastrophe at the time—and there were many besides Planck—had no a priori way of knowing whether the number of modes $n(\lambda)$ or the average energy per mode kT (or both) was the source of the problem. Both were correct classically. Many attempts were made to rederive each so as to solve the problem. It was the average energy per mode (that is, kinetic theory) that turned out to be at fault.

Classically, the electromagnetic waves in the cavity are produced by accelerated electric charges in the walls of the cavity vibrating like simple harmonic oscillators. Recall that the radiation emitted by such an oscillator has the same frequency as the oscillator itself. The average energy for a one-dimensional simple harmonic oscillator is calculated classically from the energy distribution function, which in turn is found from the Maxwell-Boltzmann distribution function. The energy distribution function has the form (see Chapter 8)

$$f(E) = Ae^{-E/kT} \qquad \textbf{3-26}$$

where A is a constant and $f(E)$ is the fraction of the oscillators with energy equal to E. The average energy is then found, as is any weighted average, from

$$\overline{E} = \int_0^\infty Ef(E)dE = \int_0^\infty EAe^{-E/kT}dE \qquad \textbf{3-27}$$

with the result $\overline{E} = kT$, as was used by Rayleigh and others.

Planck found that he could derive his empirical function by calculating the average energy $\overline{E}$ assuming the energy of the oscillating charges, and hence the radiation that they emitted, was a discrete variable, i.e., that it could take on only the values 0, ϵ, 2ϵ, . . . , $n\epsilon$ where n is an integer; and further, that ϵ was proportional to the frequency of the oscillators and, thus, the radiation. Planck therefore wrote the energy as

$$E_n = n\epsilon = nhf \qquad n = 0, 1, 2, \ldots \qquad \textbf{3-28}$$

where h is a constant now called *Planck's constant*. The Maxwell-Boltzmann distribution law (Equation 3-26) then becomes

$$f_n = Ae^{-E_n/kT} = Ae^{-n\epsilon/kT} \qquad \textbf{3-29}$$

where A is determined by the normalization condition that the sum of all fractions f_n must, of course, be 1, i.e.,

$$\sum_{n=0}^{\infty} f_n = A \sum_{n=0}^{\infty} e^{-n\epsilon/kT} = 1 \qquad \text{3-30}$$

The average energy of an oscillator is then given by the discrete-sum equivalent of Equation 3-27,

$$\overline{E} = \sum_{n=0}^{\infty} E_n f_n = \sum_{n=0}^{\infty} E_n A e^{-E_n/kT} \qquad \text{3-31}$$

Calculating the sums in Equations 3-30 and 3-31 (see Problem 3-58) yields the result:

$$\overline{E} = \frac{\epsilon}{e^{\epsilon/kT} - 1} = \frac{hf}{e^{hf/kT} - 1} = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1} \qquad \text{3-32}$$

Multiplying this result by the number of oscillators per unit volume in the interval $d\lambda$ given by Equation 3-23, we obtain for the energy density distribution function of the radiation in the cavity:

$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1} \qquad \text{3-33}$$

This function, called *Planck's law,* is sketched in Figure 3-9. It is clear from the figure that the result fits the data quite well.

For very large λ, the exponential in Equation 3-33 can be expanded using $e^x \approx 1 + x + \ldots$ for $x \ll 1$ (see Appendix B4), where $x = hc/\lambda kT$. Then

$$e^{hc/\lambda kT} - 1 \approx \frac{hc}{\lambda kT}$$

and

$$u(\lambda) \longrightarrow 8\pi\lambda^{-4} kT$$

which is the Rayleigh-Jeans formula. For short wavelengths, we can neglect the 1 in the denominator of Equation 3-33, and we have

$$u(\lambda) \longrightarrow 8\pi hc\lambda^{-5} e^{-hc/\lambda kT} \longrightarrow 0$$

as $\lambda \rightarrow 0$. The value of the constant in Wien's displacement law also follows from Planck's law, as you will show in Problem 3-23.

The value of Planck's constant, h, can be determined by fitting the function given by Equation 3-33 to the experimental data, although direct measurement (see Section 3-3) is better, but more difficult. The presently accepted value is

$$\begin{aligned} h &= 6.626 \times 10^{-34}\ \text{J}\cdot\text{s} \\ &= 4.136 \times 10^{-15}\ \text{eV}\cdot\text{s} \end{aligned} \qquad \text{3-34}$$

Planck tried at length to reconcile his treatment with classical physics but was unable to do so. The fundamental importance of the quantization assumption implied by Equation 3-28 was suspected by Planck and others but was not generally appreciated until 1905. In that year Einstein applied the same ideas to explain the photoelectric effect and suggested that, rather than being merely a mysterious property of oscillators in the cavity walls and blackbody radiation, quantization is a fundamental characteristic of light energy.

EXAMPLE 3-3 Peak of the Solar Spectrum The surface temperature of the sun is about 5800 K, and measurements of the sun's spectral distribution show that it radiates very nearly like a blackbody, deviating mainly at very short wavelengths. Assuming that the sun radiates like a perfect blackbody, at what wavelength does the peak of the solar spectrum occur?

Solution

1. The wavelength at the peak, or maximum intensity, of a perfect blackbody spectrum is given by Equation 3-20:

$$\lambda_m T = 2.898 \times 10^{-3}\ \mathrm{m\cdot K}$$

2. Rearranging and substituting the sun's surface temperature yields:

$$\lambda_m = (2.898 \times 10^{-3}\ \mathrm{m\cdot K})/T = \frac{2.898 \times 10^{-3}\ \mathrm{m\cdot K}}{5800\ \mathrm{K}}$$

$$= \frac{2.898 \times 10^{6}\ \mathrm{nm\cdot K}}{5800\ \mathrm{K}} = 499.7\ \mathrm{nm}$$

where

$$1\ \mathrm{nm} = 10^{-9}\ \mathrm{m}$$

Remarks: *This value is near the middle of the visible spectrum.*

The electromagnetic spectrum emitted by incandescent bulbs is a common example of blackbody radiation, the amount of visible light being dependent on the temperature of the filament. Another application is the pyrometer, a device that measures the temperature of a glowing object, such as molten metal in a steel mill.

EXAMPLE 3-4 Average Energy of an Oscillator What is the average energy $\overline{E}$ for an oscillator that has a frequency given by $hf = kT$ according to Planck's calculation?

Solution

From Equation 3-32 with $\epsilon = hf = kT$, we have

$$\overline{E} = \frac{\epsilon}{e^{\epsilon/kT} - 1} = \frac{kT}{e^{1} - 1} = 0.582\ kT$$

Recall that, according to classical theory, $\overline{E} = kT$ regardless of the frequency.

EXAMPLE 3-5 Stefan-Boltzmann from Planck Show that the total energy density in a blackbody cavity is proportional to T^4 in accordance with the Stefan-Boltzmann law.

Solution

The total energy density is obtained from the distribution function (Equation 3-33) by integrating over all wavelengths:

$$U = \int_0^\infty u(\lambda)d\lambda = \int_0^\infty \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1} d\lambda$$

Define the dimensionless variable $x = hc/\lambda kT$. Then $dx = -hc\, d\lambda/\lambda^2 kT$ or $d\lambda = -\lambda^2(kT/hc)dx$. Then

$$U = -\int_0^\infty \frac{8\pi hc\lambda^{-3}}{e^x - 1}\left(\frac{kT}{hc}\right) dx$$
$$= 8\pi hc\left(\frac{kT}{hc}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1} dx$$

Since the integral is now dimensionless, this shows that U is proportional to T^4. The value of the integral can be obtained from tables; it is $\pi^4/15$. Then $U = (8\pi^5 k^4/15\, h^3 c^3)\, T^4$. This result can be combined with Equations 3-19 and 3-21 to express Stefan's constant σ in terms of π, k, h, and c (see Problem 3-13).

A dramatic example of an application of Planck's law on the current frontier of physics is in tests of the predictions of the so-called Big Bang theory of the formation and present expansion of the universe. Current cosmological theory suggests that the universe originated in an extremely high-temperature explosion, one consequence of which was to fill the infant universe with radiation whose spectral distribution must surely have been that of a blackbody. Since that time, the universe has expanded to its present size and cooled to its present temperature T_{now}. However, it

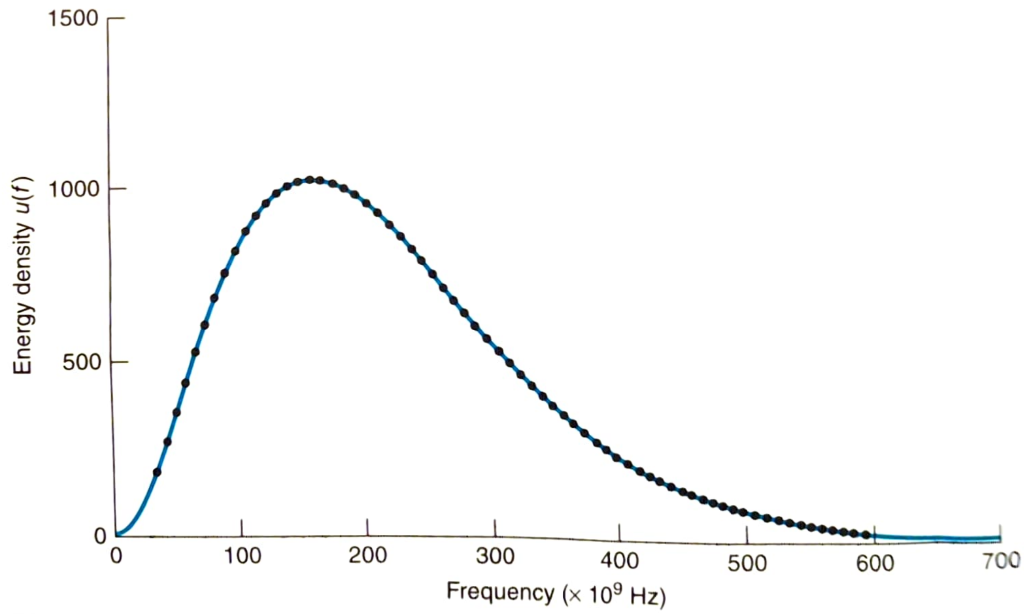

Fig. 3-10 The energy density spectral distribution of the cosmic microwave background radiation. The solid line is Planck's law with $T = 2.735$ K. The measurements were made by the COBE satellite.

should still be filled with radiation whose spectral distribution should be that characteristic of a blackbody at T_{now}.

In 1965, Arno Penzias and Robert Wilson discovered radiation of wavelength 7.35 cm reaching Earth with the same intensity from all directions in space. It was soon recognized that this radiation could be a remnant of the Big Bang fireball, and measurements were subsequently made at other wavelengths in order to construct an experimental energy density $u(\lambda)$ versus λ graph. The most recent data, collected by the Cosmic Background Explorer (COBE) satellite and shown in Figure 3-10, fit the Planck law for a blackbody at 2.735 K. The excellent agreement of the data with Planck's equation, indeed, the best fit that has ever been measured, is considered to be very strong support for the Big Bang theory (see Chapter 14).

3-3 The Photoelectric Effect

It is one of the ironies in the history of science that in the famous experiment of Heinrich Hertz[12] in 1887 in which he produced and detected electromagnetic waves, thus confirming James Clerk Maxwell's wave theory of light, he also discovered the photoelectric effect that led directly to the particle description of light.

Hertz was using a spark gap in a tuned circuit to generate the waves and another similar circuit to detect them. He noticed accidentally that when the light from the generating gap was shielded from the receiving gap, the receiving gap had to be made shorter to allow the sparks to pass. Light from any spark that fell on the terminals of the gap facilitated the passage of the sparks. He described the discovery with these words:

> In a series of experiments on the effects of resonance between very rapid electric oscillations that I carried out and recently published, two electric sparks were produced by the same discharge of an induction coil, and therefore simultaneously. One of these sparks, spark *A*, was the discharge spark of the induction coil, and served to excite the primary oscillation. The second, spark *B*, belonged to the induced or secondary oscillation. I occasionally enclosed spark *B* in a dark case so as to make observations more easily, and in so doing I observed that the maximum spark length became decidedly smaller inside the case than it was before.[13]

Albert A. Michelson, Albert Einstein, and Robert A. Millikan at a meeting in Pasadena, California, in 1931. [*AP/Wide World Photos.*]