Modern Physics Problem Set 3

Due: Sep 23, 2025

Problem 1

Show that the most direct world line for two events with a lightlike separation has the longest proper time of all possible world lines.

Hint: consider a few simple world lines as in the discussion at the end of section 6 (TW pp. 34–35).

Problem 2

Calculate β and $\gamma = (1 - \beta^2)^{-1/2}$ for the following:

1. A car traveling 100 km/h
2. A commercial jet airliner traveling 290 m/s
3. A supersonic airliner traveling at Mach 2.3
4. The space shuttle, traveling at 27,000 km/h
5. An electron traveling 25 cm in 2 ns
6. A proton traveling across a nucleus (10^{-14} m) in 0.35×10^{-22} s

Problem 3

Make a plot of $\gamma = (1 - \beta^2)^{-1/2}$ vs β. Select an appropriate range for β and use whatever technique you feel is best (paper and pencil on graph paper, computer, etc.).

Problem 4

Most elementary particles are unstable: they disintegrate after a characteristic lifetime that varies from one species to the next (e.g., lifetime of a neutron is 15 min, of a muon, 2×10^{-6} s). These are the lifetimes of particles at rest. When particles are moving at speeds close to the speed of light c, their lifetimes are much longer, since their internal clocks (whatever it is that tells them when their time is up) are running slow, in accordance with time-dilation. As an example, consider a muon that is traveling through the laboratory at three-fifths the speed of light. What is its lifetime measured in the lab?

Problem 5

A Lincoln Continental is twice as long as a VW Beetle, when they are at rest. As the Continental overtakes the VW, going through a speed trap, a (stationary) policeman observes that they both have the same length. The VW is going at half the speed of light. How fast is the Lincoln going? Leave your answer as a multiple of c.

Problem 6

Two events occur in an inertial reference system K as follows:

$$\begin{array}{lllll} \text{Event 1:} & x_1 = a, & t_1 = \frac{2a}{c}, & y_1 = 0, & z_1 = 0 \\ \text{Event 2:} & x_2 = 2a, & t_2 = \frac{3a}{2c}, & y_2 = 0, & z_2 = 0 \end{array}$$

Notice that since a is a length, presumably in meters, these times are in seconds.

1. In what frame K' will these events occur at the same time?
2. What are the coordinates of these events in that frame?
3. Calculate the interval squared in the two frames and verify that they are the same.