IN QM, THE RESULT OF AN EXPERIMENT (MEASUREMENT) DO NOT TELL US WHAT THE STATE OF A PARTICLE WAS

WE GET $\frac{3L}{4}$ FOR POSITION — WAS IT IN ψ_1 OR ψ_2 OR ...

WE MUST AVERAGE MANY MEASUREMENTS

$$\langle f(x) \rangle = \int_{-\infty}^{\infty} \psi^* f(x) \psi \, dx$$

$f(x)$ CAN INCLUDE OPERATORS, SUCH AS

$$p = -i\hbar \frac{\partial}{\partial x}$$

$$E = i\hbar \frac{\partial}{\partial t} \quad \text{OR} \quad -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$$

$$\langle p^2 \rangle = \int_0^L \psi^* \left(-i\hbar \frac{d}{dx}\right)\left(-i\hbar \frac{d}{dx}\right)\psi \, dx$$

$$= \int_0^L \left(\frac{2}{L}\right)^{1/2} \sin\left(\pi \frac{x}{L}\right) \left(-\hbar^2 \frac{d^2}{dx^2}\right) \left(\frac{2}{L}\right)^{1/2} \sin\left(\pi \frac{x}{L}\right) dx$$

$$\Rightarrow \quad \frac{d^2}{dx^2}\psi = \frac{d^2}{dx^2}\left(\frac{2}{L}\right)^{1/2} \sin\left(\pi \frac{x}{L}\right)$$

$$= \left(\frac{2}{L}\right)^{1/2} \frac{d}{dx} \cos\left(\pi \frac{x}{L}\right)\left(\frac{\pi}{L}\right)$$

$$= \left(\frac{2}{L}\right)^{1/2} \left(-\sin\left(\pi \frac{x}{L}\right)\right)\left(\frac{\pi}{L}\right)^2$$

$$= -\left(\frac{\pi}{L}\right)^2 \left(\frac{2}{L}\right)^{1/2} \sin\left(\pi \frac{x}{L}\right)$$

$$= -\left(\frac{\pi}{L}\right)^2 \psi$$

$$\langle p^2 \rangle = -\hbar^2 \int_0^L \underbrace{\left(\frac{2}{L}\right)^{1/2} \sin\left(\pi \frac{x}{L}\right)}\left(-\frac{\pi^2}{L^2}\right)\underbrace{\left(\frac{2}{L}\right)^{1/2} \sin\left(\pi \frac{x}{L}\right)} dx$$

$$= \frac{\pi^2 \hbar^2}{L^2} \underbrace{\int_0^L \psi^* \psi \, dx}_{=1} = \boxed{\frac{\pi^2 \hbar^2}{L^2} = \langle p^2 \rangle}$$