MULTIPLE CHOICE

1) - b

2) - a

3) - c

4) - a

1) $\Psi(x,t) = Ae^{kx-\omega t}$

$$i\hbar \frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi + V\Psi$$

$$\frac{\partial}{\partial t}\Psi = -\omega A e^{kx-\omega t} = -\omega \Psi$$

$$\frac{\partial}{\partial x}\Psi = kAe^{kx-\omega t} = k\Psi$$

$$\frac{\partial^2}{\partial x^2}\Psi = k^2 A e^{kx-\omega t} = k^2\Psi$$

$$+i\hbar\omega \cancel{\Psi} = \left(+\frac{\hbar^2 k^2}{2m} + V\right)\cancel{\Psi}$$

$$i\hbar\omega = \frac{\hbar^2 k^2}{2m} + V$$

$$\underbrace{iE}_{\text{IMAGINARY}} = \underbrace{\frac{p^2}{2m} + V}_{\text{REAL}} \quad \times$$

CAN'T MAKE EQUAL!

2) $V(x) = V_0$

a) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V_0\psi = E\psi$

$E < V_0$

$$\frac{d^2\psi}{dx^2} = -\frac{2m(E-V_0)}{\hbar^2}\psi = \frac{2m(V_0-E)}{\hbar^2}\psi$$

b) SHOW $\psi(x) = Ce^{\kappa x} + De^{-\kappa x}$

$$\frac{d\psi}{dx} = \kappa Ce^{\kappa x} - \kappa De^{-\kappa x} = \kappa\psi$$

$$\frac{d^2\psi}{dx^2} = \kappa^2 Ce^{\kappa x} + \kappa^2 De^{-\kappa x}$$

$$= \kappa^2(Ce^{\kappa x} + De^{-\kappa x})$$

$$= \kappa^2\psi$$

$$\kappa^2\psi = \frac{2m(V_0-E)}{\hbar^2}\psi$$

$$\kappa = \frac{(2m(V_0-E))^{1/2}}{\hbar}$$

c) ψ & ψ' CONT.

3)

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$$

$$\psi = A\sin(kx)$$

$$\psi' = kA\cos(kx)$$

$$\psi'' = -k^2A\sin(kx)$$

$$k = \frac{(2mE)^{1/2}}{\hbar}$$

$$\psi(0) = 0 = A\sin(k0) \quad \checkmark$$

$$\psi(L) = 0 = A\sin(kL) \Rightarrow kL = n\pi$$

$$k = \frac{n\pi}{L}$$

$$\psi_n = A\sin\left(\frac{n\pi}{L}x\right)$$

$$\boxed{\int \sin^2(ax) = \frac{x}{2} - \frac{\sin(2ax)}{4a} + C}$$

NORM

$$1 = \int_0^L A^2\sin^2\left(\frac{n\pi}{L}x\right)dx$$

$$= A^2\left[\frac{L}{2} - \cancel{\frac{\sin\left(\frac{2n\pi}{L}x\right)^{0}}{4L}}\,n\pi\right]$$

$$1 = A^2\frac{L}{2} \Rightarrow A = \left(\frac{2}{L}\right)^{1/2}$$

3) CONT

$$\psi_n = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{n\pi}{L}x\right) \qquad n = 1, 2, 3 \ldots$$

$$k = \frac{n\pi}{L} \; ; \quad k = \frac{(2mE)^{1/2}}{\hbar}$$

I COULDN'T REMEMBER EQUATION, SO I DERIVED.

$$\frac{n\pi}{L} = \frac{(2mE)^{1/2}}{\hbar}$$

OK IF IT IS ON YOUR EQUATION SHEET!

$$\frac{n^2\pi^2}{L^2} = \frac{2mE}{\hbar^2} \Rightarrow \boxed{E_n = \frac{n^2\hbar^2\pi^2}{2mL^2}}$$

b) for e^-

$$E_1 = \frac{\hbar^2 c^2 \pi^2}{2mc^2L^2} = \frac{(197\,eV\cdot nm)^2\pi^2}{2(511\times10^3\,eV)(0.5\,nm)^2}$$

$$= 1.5\,eV$$

c) for p

$$E_1 = 8.2\times10^{-4}\,eV$$

4) $\psi(x) = Ax(a-x) \qquad 0 < x < a$

a) Find A

$$1 = \int_{-\infty}^{\infty} \psi^* \psi = \int_0^a A^2 (xa - x^2)^2 \, dx$$

$$= A^2 \int_0^a (a^2x^2 + x^4 - 2ax^3) \, dx$$

$$= A^2 \left[\frac{a^2x^3}{3} + \frac{x^5}{5} - \frac{2ax^4}{4} \right]_0^a$$

$$= A^2 \left[\frac{a^5}{3} + \frac{a^5}{5} - \frac{2a^5}{4} \right]$$

$$= A^2 a^5 \left(\frac{1}{3} + \frac{1}{5} - \frac{1}{2} \right) = A^2 a^5 \left(\frac{8}{15} - \frac{1}{2} \right)$$

$$= A^2 a^5 \left(\frac{16}{30} - \frac{15}{30} \right) = \frac{A^2 a^5}{30} = 1$$

$$A = \left(\frac{30}{a^5} \right)^{1/2}$$

4) b) compute $\langle x \rangle$

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^* x \psi \, dx$$

$$= \int_0^a \left(\frac{30}{a^5}\right)^{1/2} x(a-x) \, x \left(\frac{30}{a^5}\right)^{1/2} x(a-x) \, dx$$

$$= \frac{30}{a^5} \int_0^a x^3 (a-x)^2 \, dx$$

$$= \frac{30}{a^5} \int_0^a x^3 (a^2 + x^2 - 2ax) \, dx$$

$$= \frac{30}{a^5} \int_0^a (a^2 x^3 + x^5 - 2ax^4) \, dx$$

$$= \frac{30}{a^5} \left[\frac{a^2 x^4}{4} + \frac{x^6}{6} - \frac{2ax^5}{5} \right]_0^a$$

$$= \frac{30}{a^5} \left[\frac{a^6}{4} + \frac{a^6}{6} - \frac{2a^6}{5} \right]$$

$$= 30a \left[\frac{1}{4} + \frac{1}{6} - \frac{2}{5} \right] = 30a \left[\frac{10}{24} - \frac{2}{5} \right]$$

$$= 30a \left[\frac{50 - 48}{5(24)} \right] = 30a \left[\frac{2}{120} \right]$$

$$= \boxed{\frac{a}{2} = \langle x \rangle}$$

4) c) PROB $0 < x < a/2$

$$P = \int_0^{a/2} \psi^* \psi \, dx = \int_0^{a/2} \frac{30}{a^5} \left(x(a-x)\right)^2 dx$$

$$= \frac{30}{a^5} \int_0^{a/2} (ax - x^2)^2 dx$$

$$= \frac{30}{a^5} \int_0^{a/2} (a^2x^2 + x^4 - 2ax^3) dx$$

$$= \frac{30}{a^5} \left[\frac{a^2x^3}{3} + \frac{x^5}{5} - \frac{2ax^4}{4} \right]_0^{a/2}$$

$$= \frac{30}{a^5} \left[\frac{a^5}{8 \cdot 3} + \frac{a^5}{32 \cdot 5} - \frac{2a^5}{16 \cdot 4} \right]$$

$$= 30 \left[\frac{1}{60} \right] = \boxed{\frac{1}{2} = P_{0<x<a/2}}$$

5) e^- IN WELL, $n=5$ $\Rightarrow \psi = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{5\pi}{L}x\right)$

a) FIND $P_{0.2L < x < 0.4L}$

$$P = \int_{0.2L}^{0.4L} \psi^* \psi \, dx = \int_{0.2L}^{0.4L} \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{5\pi}{L}x\right) \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{5\pi}{L}x\right) dx$$

$$= \int_{0.2L}^{0.4L} \frac{2}{L} \sin^2\left(\frac{5\pi}{L}x\right) dx$$

$$= \frac{2}{L} \int_{0.2L}^{0.4L} \sin^2\left(\frac{5\pi}{L}x\right) dx$$

$$= \frac{2}{L} \left[\frac{x}{2} - \frac{\sin\left(\frac{10\pi}{L}x\right) L}{4 \cdot 5\pi} \right]_{0.2L = \frac{2}{10}L}^{0.4L = \frac{4}{10}L}$$

$$= \frac{2}{L} \left[\frac{4L}{20} - \cancel{\frac{\overset{0}{\sin\left(\frac{10\pi 4L}{10L}\right)}}{4.5\pi}} - \frac{2L}{20} + 0 \right]$$

SIMILAR TO FIRST sin TERM

$$= \frac{2}{\cancel{L}} \left[\frac{4\cancel{L}}{20} - \frac{2\cancel{L}}{20} \right]$$

$$= \frac{4}{20} = \frac{1}{5}$$

5) b) $P_{x \approx \frac{L}{2}} = |\psi|^2 \Delta x$

$$= \frac{2}{L} \sin^2\left(\frac{5\pi}{L}\frac{L}{2}\right) 0.01L$$

$$= 0.02 \sin^2\left(\frac{5\pi}{2}\right)$$

$$\boxed{P = 0.02}$$

6) $\psi(x) = \left(\frac{3}{L^3}\right)^{1/2} x \qquad 0 < x < L$

a) VERIFY ψ IS NORMALIZED.

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = \int_0^L \left(\frac{3}{L^3}\right)^{1/2} x \left(\frac{3}{L^3}\right)^{1/2} x \, dx$$

$$= \frac{3}{L^3} \int_0^L x^2 \, dx = \frac{3}{L^3} \left[\frac{x^3}{3}\right]_0^L$$

$$= \frac{3}{L^3} \left[\frac{L^3}{3} - 0\right]$$

$$= 1 \quad \checkmark$$

6) b)

$$\psi(x) = \sqrt{\frac{3}{L^3}}\, x \qquad 0 < x < L$$

FIND $\langle x \rangle$

$$\langle x \rangle = \int \psi^* x \psi \, dx$$

$$= \int_0^L \left(\frac{3}{L^3}\right)^{1/2} x \,(x)\, \left(\frac{3}{L^3}\right)^{1/2} x \, dx$$

$$= \frac{3}{L^3} \int_0^L x^3 \, dx$$

$$= \frac{3}{L^3} \left[\frac{x^4}{4} \right]_0^L = \frac{3}{L^3} \left[\frac{L^4}{4} - 0 \right]$$

$$= \boxed{\frac{3}{4} L = \langle x \rangle}$$