

# Modern Physics Practice Exam — Quantum Mechanics

Time: 1.5 hours

**Instructions:** Show your reasoning clearly and include units. To receive full credit, responses must begin with an equation (or equations) from lecture or the text in symbolic form ( $\sim 1/4$  credit), followed by clear algebra leading to an answer to the question in symbolic form ( $\sim 1/2$  credit), and finally a numeric result ( $\sim 1/4$  credit).

## Part A: Multiple choice (20 points)

Choose the best answer. Each question is worth 5 points. You may circle answer on exam.

1. The normalization condition for a wavefunction  $\psi(x, t)$  in one dimension is

a)  $\int_{-\infty}^{\infty} \psi(x, t) dx = 1$

b)  $\int_{-\infty}^{\infty} |\psi(x, t)|^2 dx = 1$

c)  $\int_{-\infty}^{\infty} |\psi(x, t)| dx = 1$

d)  $\int_{-\infty}^{\infty} \psi^2(x, t) dx = 1$

2. The expectation value of momentum  $\langle p \rangle$  in one dimension is

a)  $\int \psi^*(x, t) (-i\hbar) \frac{d}{dx} \psi(x, t) dx$

b)  $\int |\psi(x, t)|^2 dx$

c)  $\int p |\psi(x, t)|^2 dp$

d)  $\int \psi(x, t) \frac{d\psi}{dx} dx$

3. Bound states in a finite square well occur only when:

a)  $E > V_0$

b)  $E = V_0$

c)  $E < V_0$

d)  $E$  is negative regardless of the well parameters

4. The quantity

$$k = \frac{\sqrt{2m(E - V)}}{\hbar}$$

in the time-independent Schrödinger equation is

- a) real for  $E > V$  and imaginary for  $E < V$
- b) real for  $E < V$  and imaginary for  $E > V$
- c) always real for any  $E$
- d) always imaginary for any  $E$

## Part B: Problems (80 points)

1. (15 pts) **Time-dependent Schrödinger Equation**

Show that the wave function  $\Psi(x, t) = Ae^{kx - \omega t}$  does not satisfy the time-dependent Schrödinger wave equation.

2. (15 pts) **Time-independent Schrödinger Equation in a region of constant potential**

Consider a region of space where the potential is constant,  $V(x) = V_0$ , and  $E < V_0$ .

- a) Write down the time-independent Schrödinger equation in this region.
- b) Show that in this region the general solution can be written as

$$\psi(x) = C e^{\kappa x} + D e^{-\kappa x}$$

and find an expression for  $\kappa$  in terms of  $m$ ,  $E$ ,  $V_0$ , and  $\hbar$ .

- c) Briefly state the boundary conditions that  $\psi(x)$  and its derivative must satisfy at a discontinuity in the potential (for example, at the edge of a finite well).

3. (10 pts) **Infinite well energies**

A particle of mass  $m$  is in an infinite square well of width  $L$  with walls at  $x = 0$  and  $x = L$ .

- a) Write the general expression for the allowed energy levels  $E_n$  for this well.
- b) Calculate the ground-state energy  $E_1$  for an **electron** in a well of width  $L = 0.5$  nm.
- c) Calculate the ground-state energy  $E_1$  for a **proton** in a well of the same width  $L = 0.5$  nm. Comment briefly on the difference between the electron and proton energies.

4. (15 pts) **Normalization and expectation values**

A particle is confined to the region  $0 \leq x \leq a$  with wavefunction

$$\psi(x) = \begin{cases} Ax(a-x), & 0 \leq x \leq a, \\ 0, & \text{otherwise.} \end{cases}$$

- Determine the normalization constant  $A$ .
- Compute the expectation value  $\langle x \rangle$ .
- Compute the probability that the particle is found in the left half of the well,  $0 \leq x \leq a/2$ .

5. (15 pts) **Probability in a small region of an infinite well**

An electron moving in a one-dimensional infinite square well is trapped in the  $n = 5$  state.

- Show that the probability of finding the electron between  $x = 0.2L$  and  $x = 0.4L$  is  $1/5$ .
- Compute the probability that the electron is in the “volume”  $\Delta x = 0.01L$  at  $x = L/2$ .

6. (10 pts) **Expectation values**

A particle in one dimension is described (for  $0 \leq x \leq L$ ) by the normalized wavefunction

$$\psi(x) = \sqrt{\frac{3}{L^3}}x$$

The wavefunction is zero outside the interval  $0 \leq x \leq L$ .

- Verify that  $\psi(x)$  is normalized.
- Compute the expectation value  $\langle x \rangle$ .

## Constants

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$$

$$\hbar = 1.05 \times 10^{-34} \text{ J} \cdot \text{s} = 6.58 \times 10^{-16} \text{ eV} \cdot \text{s}$$

$$m_e = 511 \times 10^3 \text{ eV} = 9.11 \times 10^{-31} \text{ kg}$$

$$m_p = 940 \times 10^6 \text{ eV} = 1.675 \times 10^{-27} \text{ kg}$$

$$m_n = 938 \times 10^6 \text{ eV} = 1.673 \times 10^{-27} \text{ kg}$$

$$r_{\text{sun}} = 6.96 \times 10^8 \text{ m}$$

$$\int \sin^2(ax) dx = \frac{x}{2} - \frac{\sin(2ax)}{4a} + C$$