Modern Physics Practice Exam

Time: 1.5 hours

Instructions: Answer all questions. Show your reasoning clearly and include units. Calculators are allowed.

Part A: Short Answer (20 points)

1. (4 pts) What was Max Planck's hypothesis to solve the ultraviolet catastrophe of the Rayleigh-Jeans equation?
2. (4 pts) What was Einstein's interpretation of Planck's hypothesis that he used to derive his equation for the photoelectric effect?
3. (4 pts) Einstein's photoelectric equation is a statement of what fundamental physical principle?
4. (4 pts) State Heisenberg's uncertainty principle and explain its implication for simultaneous measurements of position and momentum.
5. (4 pts) Briefly explain how Compton scattering supports the particle nature of light.

Part B: Problems (80 points)

6. (15 pts) **Blackbody Radiation**

   A blackbody laboratory source emits radiation that peaks at a wavelength of $\lambda_{\text{max}} = 520\,\text{nm}$.

   a) (5 pts) Use Wien's displacement law to determine the temperature of the source.

   b) (5 pts) The source has an emitting area of $8.0\,\text{cm}^2$. Use the Stefan–Boltzmann law to compute the total power output.

   c) (5 pts) Describe qualitatively how the spectrum changes if the temperature is doubled. Your answer should reference both total radiated power and the shape of the spectrum.

7. (15 pts) **Photoelectric Effect**

   Ultraviolet light with wavelength $\lambda = 320\,\text{nm}$ strikes a metal surface with work function $\phi = 2.8\,\text{eV}$.

   a) (5 pts) Compute the maximum kinetic energy of the emitted electrons in electron volts.

   b) (5 pts) Determine the stopping potential needed to halt the most energetic electrons.

c) (5 pts) If the intensity of the light is doubled but the wavelength is unchanged, what happens to the photocurrent and the stopping potential? Explain.

8. (10 pts) **Compton Scattering**

   An X-ray photon of wavelength $\lambda = 0.150\,\text{nm}$ undergoes Compton scattering from a stationary electron.

   a) (5 pts) Calculate the scattered wavelength at $\theta = 45°$.

   b) (5 pts) Determine the energy (in keV) of the scattered photon.

9. (10 pts) **de Broglie Waves and Uncertainty**

   a) (5 pts) A proton has speed $v = 1.5 \times 10^5\,\text{m/s}$. Compute its de Broglie wavelength.

   b) (5 pts) An electron is confined in a region of width $\Delta x = 2.0 \times 10^{-10}\,\text{m}$. Use the Heisenberg uncertainty principle to estimate the minimum kinetic energy of the electron.

10. (10 pts) **Matter Waves and Velocities**

    a) (5 pts) Compute the de Broglie wavelength of a 50 eV electron.

    b) (5 pts) For a free particle with dispersion relation $\omega = \frac{\hbar k^2}{2m}$, compute the phase and group velocities.

11. (20 pts) **Bohr Model and Rydberg Equation**

    a) (10 pts) Starting from the Bohr quantization condition and the Coulomb force law, derive the expression for the allowed energy levels of hydrogen.

    b) (5 pts) Explain how replacing the electron mass with the reduced mass modifies the predicted spectral lines. Why is this correction necessary for high precision?

    c) (5 pts) Compute the wavelength of the $n = 4 \to n = 2$ transition in hydrogen.

Constants

$c = 3 \times 10^8 \text{m/s}$
$h = 6.63 \times 10^{-34} \text{J} \cdot s = 4.14 \times 10^{-15} \text{eV} \cdot s$
$\hbar = 1.05 \times 10^{-34} \text{J} \cdot s = 6.58 \times 19^{-16} \text{eV} \cdot s$
$b = 2.9 \times 10^{-3} \text{m} \cdot \text{K}$ (Wien's law constant)
$\sigma = 5.67 \times 10^{-8} \text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$ (Stefan-Boltzmann constant)