Modern Physics Practice Exam — Special Relativity

Time: 1.5 hours

Instructions:

- Answer all questions.
- Show all reasoning and intermediate steps for full credit.
- Unless otherwise stated, take the speed of light $c = 3.00 \times 10^8\,\mathrm{m/s}$.
- You may leave times in units of either seconds or meters

Part A: Short Answer (20 points total)

1. (4 pts) Define the dimensionless parameter β.
2. (4 pts) Define the Lorentz factor γ in terms of β
3. (4 pts) What is the invariant mass of a system, and why is it important in particle physics?
4. (4 pts) State one physical effect that arises from $\gamma > 1$.
5. (4 pts) Briefly describe what is meant by a 4-vector and give an example other than the spacetime position 4-vector.

Part B: Problems (80 points total)

6. (15 pts) **Lorentz Transformations**

   Two events occur in frame S: Event A: $x = 0\,\mathrm{m}, t = 0\,\mathrm{s}$ Event B: $x = 600\,\mathrm{m}, t = 2.5\,\mu\mathrm{s}$

   A frame S' moves with velocity $v = 0.6c$ in the +x direction relative to S.

   (a) Write down the Lorentz transformation equations relating (x, t) in S to (x', t') in S'.

   (b) Compute (x', t') for Event B.

7. (15 pts) **Time Dilation and Length Contraction** A muon has a proper lifetime of 2.2 μs. It is moving through Earth's atmosphere at $0.9c$.

   (a) How long does the muon live as measured in the Earth's frame?

   (b) In the muon's frame, what is the thickness of a 600 m layer of atmosphere it passes through?

8. (15 pts) **Energy and Momentum**

   A particle of rest mass m_0 moves with speed $0.8c$ in the $\hat{x}$ direction.

(a) Write down the components of its 4-momentum.

(b) Calculate its total energy in terms of m_0.

(c) Verify that $E^2 - p^2 = m_0^2$.

9. (15 pts) **Velocity Addition**

   A spaceship moves at $0.8c$ relative to Earth. A probe is launched from the spaceship at $0.5c$ relative to the ship (in the same direction).

   (a) What is the velocity of the probe relative to Earth?

   (b) Explain briefly why velocities do not simply add in special relativity.

10. (20 pts) **Invariant Mass of a System**

    Two photons collide head-on in the lab frame. One has energy 4 MeV moving in the $+x$ direction, the other has energy 6 MeV moving in the $-x$ direction.

    (a) Write the energy and momentum 4-vectors of each photon.

    (b) Compute the total 4-momentum of the system.

    (c) Calculate the invariant mass of the two-photon system.

Total: 100 points
Part A (20 points) Part B (80 points)