

rors. Such a device is a *clock*. This clock may be said to “tick” each time the light flash arrives back at the first mirror. Between ticks the light flash has traveled a round-trip distance of 1 meter. Therefore the unit of time between ticks of this clock is called *1 meter of light-travel time* or more simply *1 meter of time*. (Show that 1 second is approximately equal to 3×10^8 meters of light-travel time.)

One purpose of the physicist is to sort out simple relations between events. To do this here he might as well choose a particular reference frame with respect to which the laws of physics have a simple form. Now, the force of gravity acts on everything near the earth. Its presence complicates the laws of motion as we know them from common experience. In order to eliminate this and other complications, we will, in the next section, focus attention on a freely falling reference frame near the earth. In this reference frame no gravitational forces will be felt. Such a gravitation-free reference frame will be called an *inertial reference frame*. Special relativity deals with the classical laws of physics expressed with respect to an inertial reference frame.

Simplify: Pick freely falling laboratory

The principles of special relativity are remarkably simple. They are very much simpler than the axioms of Euclid or the principles of operating an automobile. Yet both Euclid and the automobile have been mastered—perhaps with insufficient surprise—by generations of ordinary people. Some of the best minds of the twentieth century struggled with the concepts of relativity, not because nature is obscure, but simply because man finds it difficult to outgrow established ways of looking at nature. For us the battle has already been won. The concepts of relativity can now be expressed simply enough to make it easy to think correctly—thus “making the bad difficult and the good easy.”† The problem of understanding relativity is no longer one of *learning* but one of *intuition*—a practiced way of seeing. When seen with this intuition, a remarkable number of otherwise incomprehensible experimental results are revealed to be perfectly natural.‡

2. The Inertial Reference Frame

Less than a month after the surrender at Appomattox ended the American Civil War (1861–65), the French author Jules Verne began writing *A Trip from the Earth to the Moon* and *A Trip around the Moon*.§ Eminent American cannon designers, so the story goes, cast a great cannon in a pit dug in the earth of Florida with the cannon muzzle pointing skyward. From this cannon is fired a 10-ton projectile containing three men and several animals. As the projectile coasts outward in unpowered flight toward the moon after leaving the cannon, its passengers walk normally inside the projectile on the side

†Einstein, in a similar connection, in a letter to the architect Le Corbusier.

‡For a comprehensive set of references to introductory literature concerning the special theory of relativity, together with several reprints of articles, see *Special Relativity Theory*, Selected Reprints, published for the American Association of Physics Teachers by the American Institute of Physics, 335 East 45th Street, New York 17, New York, 1963.

§Paperback edition published by Dover Publications, New York. Hardcover edition published in the Great Illustrated Classics Series by Dodd, Mead and Company, New York, 1962.

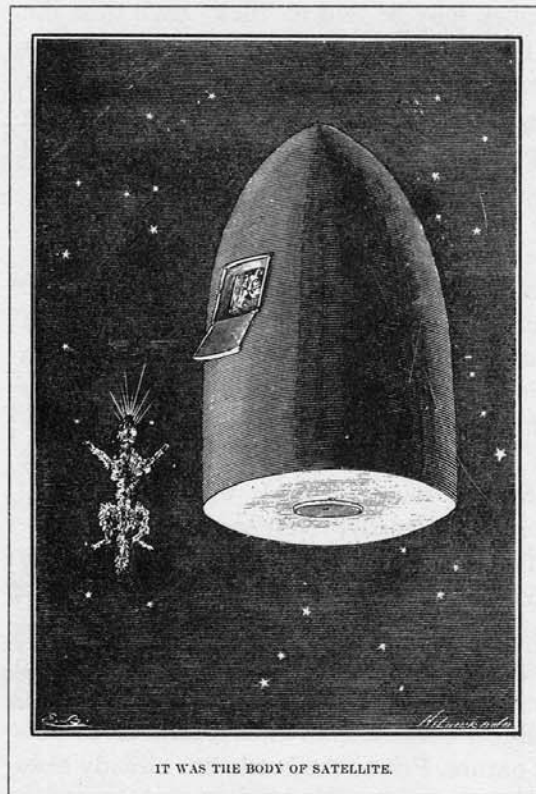


Fig. 2. Illustration from an early edition of *A Trip around the Moon*. Satellite is the name of the unfortunate dog.

*Passenger felt weight
in Jules Verne's space
ship*

nearer the earth (Fig. 3, A). As the trip continues, the passengers find themselves pressed less and less against the floor of the space ship until finally, at the point where the earth and moon exert equal but opposite gravitational attraction for all objects, the passengers float free of the floor. Later, as the ship nears the moon, they walk around once again, but now against the side of the space ship nearer the moon. Early in the trip one of the dogs in the ship had died from injuries sustained at takeoff. The passengers had disposed of the remains of the dog through a scuttle in the side of the space ship, only to find that the corpse continues to float outside the window during the entire trip.

*Paradox of passenger
and dog*

This story leads to a paradox of crucial importance to relativity. Verne thought it reasonable that the gravitational attraction of the earth would keep a passenger pressed against the earth side of the space ship during the early part of the trip. He also thought it reasonable that the dog should remain next to the ship, since both ship and dog independently follow the same path through space. But if the dog floats *outside* the space ship during the entire trip, why doesn't the passenger float around *inside* the space ship? If the ship were sawed in half would the passenger, now "outside," float free of the floor?

*Passenger is weight-
less in real space ship*

Our experience with actual space flights enables us to resolve this paradox. Jules Verne was in error about the motion of the passenger inside the space ship. Like the dog outside the ship, the passenger inside independently follows the same path through space as the space ship itself. Therefore he floats freely relative to the ship during the entire trip (Fig. 3,B). It is true that the gravitational field of the earth acts on the passenger. But it also acts on the space ship. In fact, with respect to the earth, the acceleration of the *spaceship* in the gravitational field of the earth is just *equal* to the acceleration of the

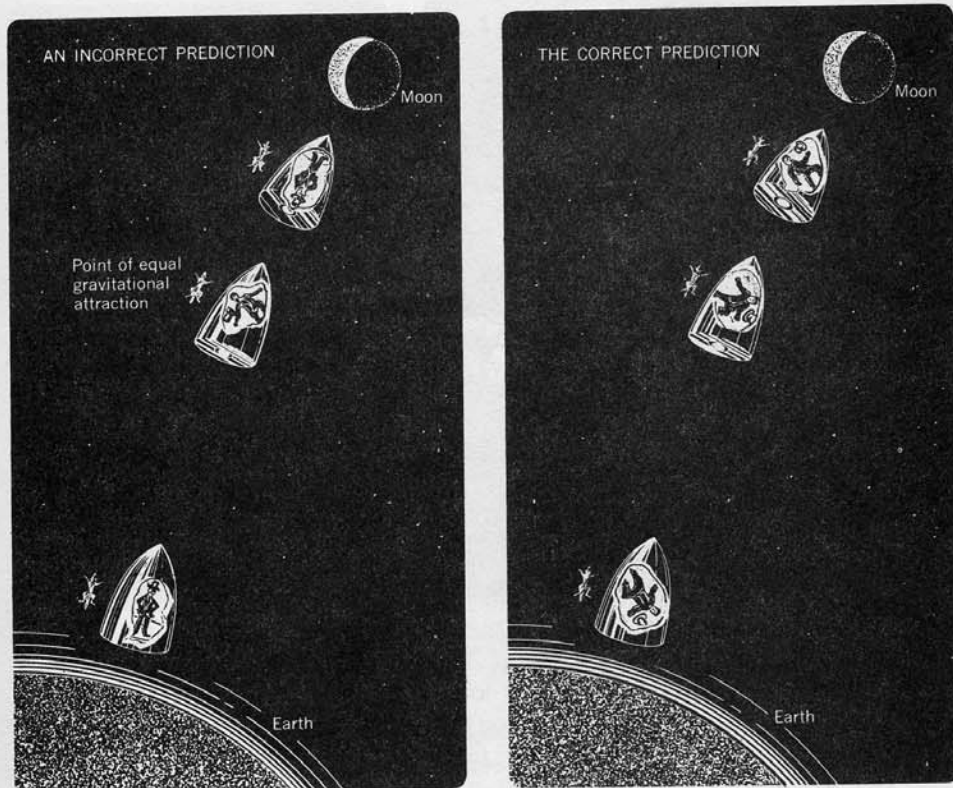


Fig. 3, A. Jules Verne believed that a passenger in a free projectile would stand on the side of the projectile nearest to the earth or moon, whichever had greatest gravitational attraction—but that the dog would float along beside the projectile during the entire trip.

Fig 3, B. Correct prediction is that a passenger would float with respect to the projectile during entire trip. Verne was correct about the motion of the dog.

passenger in the gravitational field of the earth. Because of the equality of these accelerations there will be no *relative* acceleration between passenger and space ship. Thus the space ship serves as a reference frame (“inertial reference frame”) relative to which the passenger does not experience an acceleration.

To say that the acceleration of the passenger relative to the space ship is zero is not to say that his velocity relative to it is necessarily also zero. He may have jumped from the floor or sprung from the side—in which case he will hurtle across the space and strike the opposite wall. However, when he has zero initial velocity relative to the ship the situation is particularly interesting, for he will also have zero velocity relative to it at all later times. He and the ship will follow identical paths through space. How remarkable that the passenger who cannot see the outside nevertheless moves on this deterministic orbit. Without a way to control his motion and even with his eyes closed he will not touch the wall. How could one do better at eliminating gravitational influences!

A modern space ship carrying a passenger is shot vertically from the earth, rises, and falls back toward the earth (Fig. 4). (The passenger of an elevator car experiences a close approximation to this fall when the elevator cable is cut!) Choose this freely falling space ship as the best possible reference frame in which to do physics. This reference frame is best because, among other

Concept of inertial reference frame

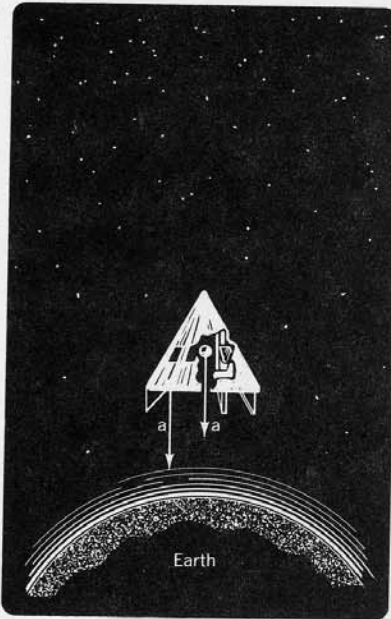


Fig. 4. Space ship in free fall near the earth.

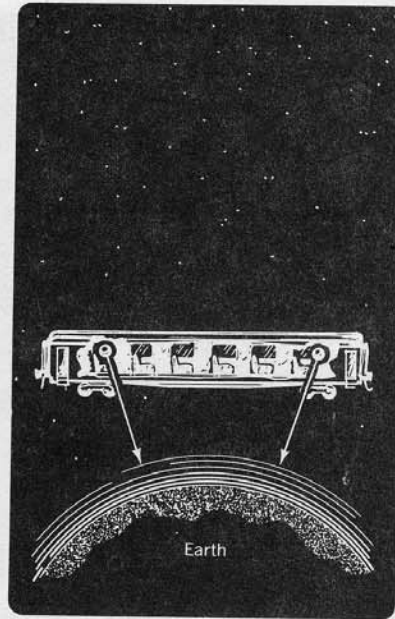


Fig. 5. Railway coach in free fall in horizontal position near the earth.

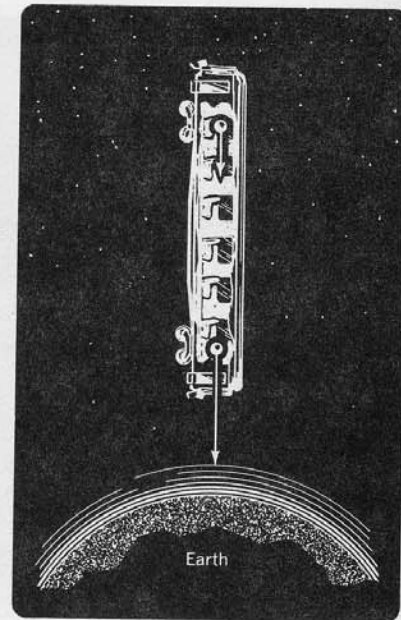


Fig. 6. Railway coach in free fall in vertical position near the earth.

things, the laws of motion of a particle are simple in a falling space vehicle. A free particle at rest in the vehicle remains at rest in the vehicle. When the particle is given a gentle push, it moves across the vehicle in a straight line with constant speed. Further experiments show that *all* the laws of mechanics can be expressed simply with respect to a falling space ship. We call such a space ship that rises or falls freely—or more generally moves freely in space—an *inertial reference frame*.

Look at the freely falling space ship from the surface of the earth. There is a simple reason why the free particle at rest relative to the space ship remains at rest in the space ship. This reason is that, with respect to the surface of the earth, the particle and the space ship both fall with the same acceleration (Fig. 4). It is because of this equal acceleration that the *relative* positions of the particle and the space ship do not change if the particle is originally at rest in the space ship.

The definition of an inertial frame requires that *no gravitational forces will be felt in it*. If such a reference frame is to be a space ship near the earth, it cannot be a very large one because widely separated particles within it will be differently affected by the *nonuniform* gravitational field of the earth. For example, particles released side by side will each be attracted toward the center of the earth, so they will move closer together as observed from the falling space ship (Fig. 5). As another example, think of the two particles being released far apart vertically but directly above one another (Fig. 6). Their gravitational accelerations toward the earth will be in the same direction. However the particle nearer the earth will slowly leave the other one behind: the two particles will move farther apart as the space ship falls. In either of these instances the laws of mechanics will not be simple in a very large space ship: the large space ship will not be an inertial frame.

Earth's pull nonuniform: large space ship is not inertial frame

Now, we want the laws of mechanics to look simple in the space ship. Therefore we want to eliminate all relative accelerations produced by external causes—"eliminate" meaning to reduce these accelerations below the limit of detection so that they will not interfere with the more important accelerations we wish to study, such as those produced when two particles collide. This can be done by choosing a space ship that is sufficiently small. The smaller the space ship, the smaller will be the relative accelerations of objects at different points in the space ship. Let someone have instruments for the detection of relative accelerations with any given degree of sensitivity. No matter how fine that sensitivity, the space ship can always be made so small that these perturbing relative accelerations are too small to be detectable. Within these limits of sensitivity the space ship is then an *inertial reference frame*.

When is a space ship or any other vehicle small enough to be called an inertial reference frame? Or when is the relative acceleration of free particles at opposite ends of the vehicle too slight to be detected? Analyzing the conditions inside one vehicle will serve to illustrate these considerations. A railway coach 25 meters long is dropped in a *horizontal* position from a height of 250 meters onto the surface of the earth (Fig. 5). The time from release to impact is about 7 seconds, or about 21×10^8 meters of light-travel time. Let tiny ball bearings be released initially from rest—and in mid-air—at opposite ends of the coach. Then, during the time of fall, they will move *toward* each other a distance of 10^{-3} meters—the thickness of 9 pages of this book—because of the difference in *direction* of the earth's gravitational pull upon them (see Ex. 32). As another example, assume that the same railway coach is dropped in a *vertical* position, and that the lower end of the coach is initially 250 meters from the surface of the earth (Fig. 6). Again two tiny ball bearings are released from rest at opposite ends of the coach. In this case, during the time of fall, the ball bearings will move *apart* by a distance of 2×10^{-3} meters because of the greater gravitational acceleration of the one nearer the earth. In either of these examples let the measuring equipment in use in the coach be just short of the sensitivity required to detect the relative motion of the ball bearings. Then, with equipment of this degree of sensitivity, and with the limited time of observation, the railway coach—or, to use an earlier example, the freely falling space ship—serves as an inertial reference frame. When the sensitivity of the measuring equipment is increased, then the space ship will not serve as an inertial reference frame unless changes are made. Either the 25-meter domain in which observations are made must be shortened, or the time given to the observations must be decreased. Or, better, some appropriate combination of the space and time dimensions of the region under observation must be cut down. Or, as a final alternative, the whole apparatus must be shot by a rocket (part c of Ex. 32) up to a region of space where one cannot detect the "differential in the gravitational acceleration" between one side of the coach and another—to use one way of speaking. In another way of speaking, the accelerations of the particles *relative to the coach* must be too small to be perceived. These relative accelerations can be measured from inside the coach without observing anything external. Only when these relative accelerations are too small to be detected is there a reference frame with respect to which the laws of motion are simple—an *inertial reference frame*.

A reference frame is said to be inertial in a certain region of space and time when, throughout that region of spacetime, and within some specified accuracy, every test particle that is initially at rest remains at rest, and every test particle that is initially in motion continues that motion without change in speed or in

Example of space ship small enough to be inertial frame

Inertial reference frame defined

direction. An inertial reference frame is also called a *Lorentz reference frame.* In terms of this definition, inertial frames are necessarily always *local* ones, that is, inertial in a limited region of spacetime.

“Region of spacetime.” What is the precise meaning of this term? The long narrow railway coach in the example served as a means to probe spacetime for a limited stretch of time and in one or another single direction in space. It can be oriented north-south, or east-west, or up-down. Whatever the orientation, the relative acceleration of the tiny ball bearings released at the two ends can be measured. For all three directions—and for all intermediate directions—let it be found by calculation that the relative drift of the two test particles is half the minimum detectable amount or less. Then throughout a cube of space 25 meters on an edge and for a lapse of time of 7 seconds, test particles moving every which way depart from straight-line motion by undetectable amounts.

*Region of spacetime
defined*

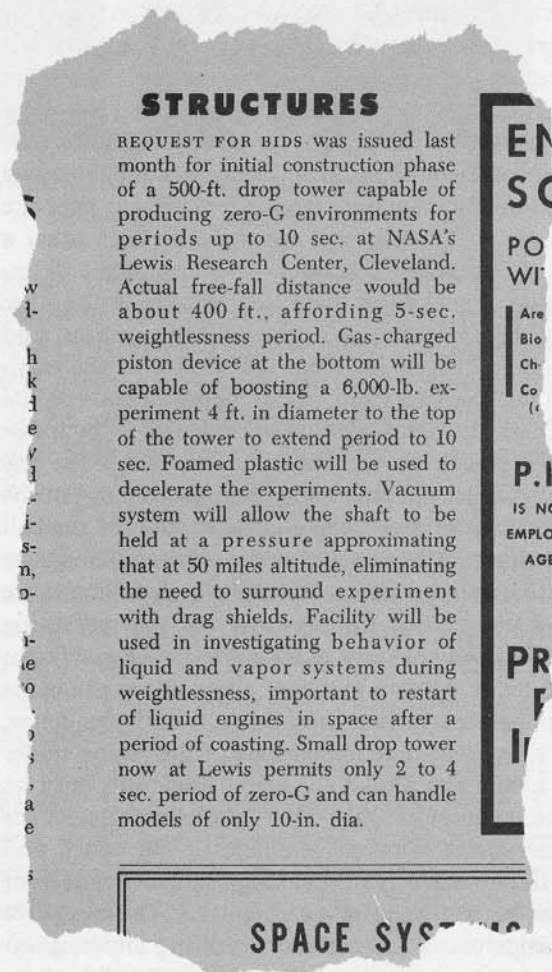


Fig. 7. Modern inertial reference frame. From *Engineering Opportunities*, March 1964.

In other words, the reference frame is inertial in a region of spacetime with dimensions

$$(25 \text{ meters} \times 25 \text{ meters} \times 25 \text{ meters of space}) \times (21 \times 10^8 \text{ meters of time})$$

For a discussion of spacetime regions larger than those of local inertial frames, see Chapter 3.

“Test particle.” How small must a particle be to qualify as a test particle? It must have so little mass that, within some specified accuracy, its presence will not affect the motion of other nearby particles. In terms of Newtonian mechanics the gravitational attraction of the test particle for other particles must be negligible within the accuracy specified. As an example, consider a particle of mass 10 kilograms. A second and less massive particle placed one-tenth meter from it and initially at rest will, in less than three minutes, undergo a displacement of 10^{-3} meters. Thus the 10-kilogram object is not—in this sense—a test particle. A test particle *responds* to gravitational forces but it does not itself *produce* any significant gravitational force.

Test particle defined

It would be impossible to define an inertial reference frame if it were not for a remarkable feature of nature. Particles of different size, shape, and material in the same location all fall with the *same acceleration* toward the earth. If this were not so, an observer inside a falling space ship would notice a relative acceleration among different particles even when they are close together; at least some of the particles initially at rest would not remain at rest; that is, the space ship would not be an inertial reference frame according to the definition. How sure are we that particles in the *same* location but of *different* substances all fall toward the earth with the same acceleration? According to legend Galileo dropped balls made of different materials from the Leaning Tower of Pisa in order to verify this assumption.† In 1922 Baron Roland von Eötvös checked to an accuracy of five parts in 10^9 that the earth imparts the same acceleration to wood and to platinum. More recently Robert H. Dicke has pointed out that the sun is more suitable than the earth as source for the gravitational acceleration that one will measure (see Ex. 35). The alternation in direction of the sun’s pull every 12 hours lends itself to fantastic amplification by resonance. Cylinders of aluminum and gold experience accelerations due to the sun (0.59×10^{-2} meters per second per second) that are the same to three parts in a hundred thousand million (3 in 10^{11}), according to R. H. Dicke and Peter G. Roll.‡ This is one of the most sensitive checks of a fundamental physical principle in all of physics: the identity of the acceleration produced by gravity in every kind of test particle.

Inertial frame is definable because all substances fall with same acceleration

It follows from this principle that a particle made of *any* material can be used as a test particle to determine whether a given reference frame is inertial. A reference frame that is inertial for one kind of test particle will be inertial for all kinds of test particles.

3. The Principle of Relativity

We describe the motion of test particles with reference to a particular reference frame in order to determine whether that frame is inertial. The same test particles and—if they collide—the same collisions may be described with reference to more than one inertial frame. The one reference frame might be carried by a space ship built like a hollow cylinder (Fig. 8,A), the other by a second craft of similar construction just enough smaller to zoom through in-

†On the question whether Galileo actually performed this experiment, see *Physics the Pioneer Science* by Lloyd W. Taylor, (Dover Publications, New York, 1959), Vol. 1, p. 25.

‡See the chapter on experimental relativity by Dicke in *Relativity, Groups, and Topology*, edited by C. and B. DeWitt, (Gordon and Breach, New York, 1964), pp. 173–177 or the book by Dicke *The Theoretical Significance of Experimental Relativity* (Gordon and Breach, New York, 1964).