

RESEARCH STATEMENT

JON P. BANNON

1. INTRODUCTION

In their groundbreaking 1936-1943 work *Rings of Operators*, F. J. Murray and John von Neumann laid the foundation for the theory of von Neumann algebras [MvN36], [MvN37], [MvN40], [MvN43]. The early motivation for the subject is found in group representation theory and in von Neumann's work on the Hilbert space formalism for quantum mechanics. The fundamental examples of von Neumann algebras are constructed using groups and their actions, and consequently the most promising lines of research in the theory pursue analogies with group theory and dynamics.

The theory of von Neumann algebras provides an especially fertile common ground for exploring rigidity phenomena for groups, group actions and Borel equivalence relations. Popa's deformation/rigidity strategy and intertwining by bimodules technique together heralded a paradigm shift in the subject and have led to a flood of striking results in recent years [Po07]. As its name suggests, the deformation/rigidity strategy elucidates the structure of von Neumann algebras that admit both a "deformable" and a "rigid" part. Two central open problems in the subject lie beyond the reach of the strategy at present: the isomorphism problems for free group factors, which are von Neumann algebras that have no substantial rigid part, and for the group factors of property (T) groups, which have no substantial deformable part. My research program is a detailed study of bimodules of von Neumann algebras, particularly of mixing properties of these, which hopefully will help bridge the gap.

Regarding the appeal of my work external to operator algebras, the theory of bimodules of von Neumann algebras powerfully generalizes both the theory of unitary representations of groups and the theory of joinings of dynamical systems, each of which admit fundamental connections with physics and number theory¹. Finding and developing such connections outside my native theory is a source of great motivation, and I have spent years cultivating relationships with many strong potential collaborators both internal and external to the theory of von Neumann algebras.

The order I've presented the research contributions below is intended to show the coherence and direction of my research program. I have discussed cited published articles as well as work in preparation. The final subsection describes my undergraduate research collaborations.

¹These include, but are not limited to, Kirillov's Orbit Method and Szemerédi's Theorem.

2. RESEARCH PROGRAM

We briefly provide further context on the theory of von Neumann algebras. A $*$ -algebra of bounded operators on a complex separable Hilbert space is a *von Neumann algebra* if it is closed in the topology determined by matrix coefficient convergence². Murray and von Neumann proved that every von Neumann algebra can be decomposed as a direct integral of von Neumann algebras each having trivial center, which are called *factors*. Intuitively, von Neumann algebras admit a “block diagonal” decomposition in which the blocks on the diagonal are parameterized by a measure space and these blocks almost everywhere are factors. For example, the spectral theorem implies that every abelian von Neumann algebra is isomorphic to some $L^\infty(X, \mu)$ acting as multiplication operators on $L^2(X, \mu)$ with some multiplicity, where (X, μ) a Borel measure space. In this case the factors in the direct integral decomposition are almost everywhere \mathbb{C} , and we may regard $L^\infty(X, \mu)$ as an algebra of “measure-theoretic diagonal matrices”. The first major surprise in the theory was that nonatomic measure-theoretic phenomena are not relegated to the diagonal, but also could occur in the blocks. We view a normalized faithful trace as analogous to a probability measure and call a factor *finite* if it admits a normalized trace. The “atomic” finite factors, called type I_n factors admit a partition of the identity by n orthogonal projections of minimal nonzero trace and are $*$ -isomorphic to the finite-dimensional matrix algebra $M_n(\mathbb{C})$. The “nonatomic” finite factors admit no nonzero projection of minimal trace and are called type II_1 factors. The trace restricted to the lattice of projections yields the dimension function of the finite factor, which measures the dimension of the range of a projection relative to the factor. Even if a factor is not finite it still admits a dimension function with range a subset of $[0, \infty]$. If the range of this dimension function is $\{0, 1, 2, \dots, \infty\}$ the factor is of type I_∞ and is isomorphic to $B(\mathcal{H})$. If the range of the dimension function is $[0, \infty]$ the factor is of type II_∞ . If the range of the dimension function is $\{0, \infty\}$ then the factor is of type III . The classification of factors reduces essentially to the type II_1 case roughly because every type III factor can be built using an action of \mathbb{R} on a type II_∞ factor, which in turn breaks down into a type II_1 and type I factor.

The theory of factors has connections to differential geometry via foliation theory, to the topology of covering spaces and index theory via the L^2 -Betti numbers originally considered by Atiyah, and the original examples of the subject bridge ergodic theory and group theory. Perhaps the most compelling external motivation for thinking about von Neumann algebras is that in the most widely accepted framework for nonrelativistic quantum mechanics as a statistical theory, in which the results of experiments are regarded as statistical pairings of states and observables, von Neumann algebras appear as the observable algebras that are appropriate if you want to ask “yes or no” questions³. Furthermore in the relativistic setting the Haag Kastler axioms are a minimal yet robust framework for quantum field theory which have

²the weak operator topology

³von Neumann algebras are generated by their orthogonal projections, which can be seen using the the spectral theorem and von Neumann’s double commutant theorem.

been shown over the past 20 years by Kawahigashi, Longo et. al. to require the local algebras of the theory to be type III_1 factors, which in turn exhibit a natural *emerging time*⁴[Pe11].

2.1. The Følner Invariant. Imagine the following magic trick. A vase falls from a shelf and shatters into finitely many pieces. The pieces are then swept into two nonempty piles. Each of the piles is then assembled into a vase identical to the original. A mathematical analogue of this trick, carried out with a countable group instead of a vase, and such that the “pieces” are assembled via left translations, is called a paradoxical decomposition of the group. By a result of Tarski, a countable group admits a paradoxical decomposition if and only if it is not *amenable*, i.e. no finitely additive probability measure on the group is invariant under left translation [Ta38]. In my dissertation, I introduced and studied a new isomorphism invariant for von Neumann algebras that quantifies the failure of amenability.

Connes’s classification of injective factors establishes a deep analogy between the amenability of groups and the injectivity of von Neumann algebras, and via this analogy establishes that there is a unique injective type II_1 factor von Neumann algebra R up to $*$ -isomorphism [Co76]. A consequence of this analogy is that the group von Neumann algebra $L\Gamma$ generated by the range of the left regular representation of Γ on $\ell^2(\Gamma)$ is injective if and only if Γ is amenable. The class of pairwise nonisomorphic amenable groups whose group von Neumann algebras are type II_1 factors is very large, and yet all of these group von Neumann algebras are $*$ -isomorphic to R . The theory encapsulates amenability into a single object.

The deep analogy referred to above is afforded by interpreting amenability of groups from the point of view of their unitary representations: a group is amenable if and only if its trivial representation is weakly contained in its left regular representation. Unitary representations of a group are (functorially) analogous to correspondences, i.e. binormal Hilbert bimodules, of the group von Neumann algebra. The notion of correspondence makes sense for type II_1 factors in general, and not just those arising from groups. The precise analogy establishes the correspondence analogues of the trivial and regular representations for a general type II_1 factor M and asserts that M is injective if and only if the correspondence analogue of the above characterization of amenability holds. We will refer to this in what follows as “the deep analogy”, since most of my research involves further exploring this analogy from various angles.

As a byproduct of the deep analogy, Følner’s condition characterizing amenable groups as those exhausted by a sequence almost invariant finite sets has an analogue for injective factors in which finite sets are replaced by finite rank projections in the ambient $B(\mathcal{H})$. The new invariant $\text{Føl}(M)$ introduced in my dissertation measures the failure of this general “Connes-Følner condition” and evaluates to 0 if and only

⁴For each faithful normal weight on a type III factor the Tomita-Takesaki theory associates a one-parameter modular automorphism group of the factor. By Connes’s cocycle theorem for different weights the modular groups are inner conjugate, hence the outer automorphism class of the modular group is unique. This is the emerging time.

if M is $*$ -isomorphic to R . We obtained various bounds on the invariant for the free group factors with Mohan Ravichandran in [BaRa07]. Recently, others have obtained sharper upper bounds for the “local version” of this invariant for the free group factors, revealing an unexpected connection with Popa’s word length deformation in deformation/rigidity theory [Ji13]. The challenge of finding explicit “local” lower bounds for the free group factors is still an interesting open problem that may benefit from a clever combinatorial attack.

2.2. The Haagerup Property. There are various fruitful weakenings of amenability that reflect the large-scale geometry and growth properties of groups. One such weakening is the Haagerup property (also known as a-T-menability), which replaces the regular unitary representation in the expression of amenability above with a C_0 -representation: a unitary representation whose matrix coefficient functions all lie in $C_0(\Gamma)$. This is a nontrivial weakening of amenability, since the nonabelian free groups all have the Haagerup property [ChCoJoJuVa01].

My work with Junsheng Fang on the Haagerup property answered two technical questions of Sorin Popa, who is the world’s leading expert on type II_1 factors [BaFa11].

The first question regarded the removal of a tracial smoothness hypothesis in the definition of the Haagerup property for finite von Neumann algebras in [Jo02]. By successfully removing this smoothness assumption, we were able to introduce a notion of C_0 -correspondence which framed the Haagerup property for finite von Neumann algebras in a way that naturally generalizes the deep analogy.

The second question asked whether the Haagerup property enjoyed a natural heredity property with respect to coamenable inclusions. The above natural notion of C_0 -correspondence provided an affirmative answer of this question, in sharp contrast with the fact that this heredity property fails for a subtly different notion of coamenable inclusion commonly used in subfactor theory.

2.3. Weakly Mixing Correspondences and Property (T). A free, ergodic probability measure-preserving action of a countable group on a standard Borel space naturally induces a unitary representation on the L^2 space of measurable functions of integral zero, called the Koopman representation. The action is mixing if and only if its Koopman representation is C_0 . Thus C_0 unitary representations are also known as mixing unitary representations. From this standpoint, our earlier work on the Haagerup property provided results about mixing correspondences.

There is a notion of weakly mixing unitary representation, which for the Koopman representation encodes weak mixing of the group action. In recent work with Jan Cameron and Stuart White, we successfully obtained a coefficient characterization of weakly mixing correspondences, advancing my earlier work with Junsheng Fang.

Although to the “naked eye” mixing properties in ergodic theory are difficult to distinguish, there is an interesting dichotomy between mixing and weakly mixing transformations. With respect to a natural topology on (ergodic) transformations Halmos proved that weakly mixing transformations are generic [Ha44], and shortly after Rokhlin proved that mixing transformations are nowhere dense [Ro48]! If instead

one considers unitary representations with Fell's topology, the Haagerup property is an obstruction to getting an analogue of Rokhlin's result, however Bergelson and Rosenblatt proved that weakly mixing unitary representations of arbitrary amenable locally compact groups are generic [BeRo88]. Subsequently, Kerr and Pichot were able to prove that for unitary representations, generic weak mixing is equivalent to the group failing to have Kazhdan's property (T) [KePi08]. In our work with Cameron and White, the precise von Neumann algebra analogue of this result is obtained [BaCaWh14]. The proof of this result resisted a straightforward approach and required delicate technical ideas. In trying to find a counterexample to vindicate some of the technical arguments needed in the paper, we sought the assistance of Stefaan Vaes. Stefaan provided us with the needed counterexample and in the process has discovered a new simplified topology on correspondences that remarkably still captures all facets of the theory thus far, from the deep analogy to the correspondence characterizations of the Haagerup property and property (T).

In addition to a natural collaboration to extend the results of [CaFaMu13] to the correspondence setting that is already underway, I intend to undertake an in-depth study of the correspondence analogue of the Fourier algebra of a discrete group. The following provides rationale for this direction. A noncommutative group von Neumann algebra $L\Gamma$ is dual to the Banach space $A(\Gamma)$ of matrix coefficients of the left regular representation⁵. Due to Fell's absorption principle, $A(\Gamma)$ is a commutative Banach algebra with respect to pointwise multiplication, called the Fourier algebra of Γ . A result of M. Walter asserts that the Banach algebra isomorphism class of the commutative Banach algebra $A(\Gamma)$ encodes the isomorphism class of the nonabelian group Γ as the spectrum of $A(\Gamma)$ [Wa72]. Contemporary attacks at rigidity problems incorporate a more modern relative of this result found in the theory of compact quantum groups. A choice of Γ generating $L\Gamma$ gives a comultiplication $\Delta_\Gamma : L\Gamma \rightarrow L\Gamma \bar{\otimes} L\Gamma (= L(\Gamma \times \Gamma))$ extending the map $\lambda_g \mapsto \lambda_g \otimes \lambda_g$, where λ is the left regular representation. The quantum group analogue of Walter's result is the following: given the comultiplication Δ_Γ , the group Γ is encoded as those elements x in $L\Gamma$ such that $\Delta_\Gamma(x) = x \otimes x$. If two comultiplications coming from different groups Γ, Λ such that $L\Gamma = L\Lambda$ are unitarily conjugate via a unitary in $L\Gamma \bar{\otimes} L\Gamma$, i.e. $\Delta_\Gamma = Ad(U) \circ \Delta_\Lambda$ then the groups must be isomorphic. Such intertwining results are completely out of reach if Γ has property (T), since in this case $\Gamma \times \Gamma$ also has property (T) and there are no deformations that can be used to obtain the intertwiner U . In spite of this, the rigidity of matrix coefficients for property (T) groups screams out that there should not be many comultiplications available! This signals that a completely new approach is needed. I will attempt to gain some traction by carefully considering the correspondence analogue of the Fourier algebra.

2.4. Noncommutative Joinings. A joining of two ergodic Γ -systems (actions on measure spaces (X, μ) and (Y, ν)) is a measure on the product σ -algebra that is invariant under the diagonal Γ -action and has marginals μ and ν . Alternatively, a joining can be viewed as a Γ -equivariant correspondence between the von Neumann

⁵The norm on $A(\Gamma)$ is given by $\|\langle \cdot, \xi, \eta \rangle\| = \inf\{\|\xi'\| \|\eta'\| : \langle \cdot, \xi \rangle = \langle \cdot, \xi', \eta' \rangle\}$.

algebras $L^\infty(X, \mu)$ and $L^\infty(Y, \nu)$. The theory of joinings in ergodic theory has its root in a question of Furstenberg asking whether two systems admit a joining that is not the product measure if and only if they share a common subsystem. D. Rudolph exhibited an example of two systems with a nontrivial joining but no common subsystem, and the accompanying construction known as “Rudolph’s Counterexample Machine” provided many short proofs of open problems in Ergodic theory. Interestingly, joining theory seemed to develop largely independently of developments in operator algebras, despite longstanding robust ties between operator algebras and ergodic theory [Gl03].

Recent work with Jan Cameron and Kunal Mukherjee undertakes an extensive and systematic study of noncommutative joinings as correspondences [BaCaMu14]. This point of view is motivated by the work of David Kerr, who suggested this direction to me during a visit to TAMU some years ago. A few papers in the literature [Du08], [Du10], [Du12] attempt to develop a theory of noncommutative joinings from a different point of view, but in these papers certain important basic results in the theory are inaccessible, e.g. that a W^* -dynamical system is disjoint from all ergodic compact systems if and only if the system is weakly mixing. The correspondence-theoretic point of view we provide of noncommutative joining as Γ -symmetric Markov map is used to obtain the above noncommutative joining-theoretic characterization of weak mixing. In fact, as a consequence of the proof of this result we obtain a rigidity result for type *III* factors that has some physical significance: given a state φ on a type *III* factor M having trivial centralizer, every φ -preserving ergodic action of a separable locally compact group on M is necessarily weakly mixing. A result of V. Jakšić and C.-A. Pillet establishes the above for actions of \mathbb{R} , but the joining point of view extends this to actions of any separable locally compact group, provided the centralizer of the state is trivial. These ideas also provide us with a new proof of the Høegh-Krohn-Landstad-Størmer Theorem: a von Neumann algebra admits an ergodic action of a compact group if and only if the factor was an injective finite von Neumann algebra [HLS81].

The noncommutative theory of joinings brings together two powerful $*$ -isomorphism invariants of a von Neumann algebra, its space of correspondences and its automorphism group, and also provides a new connection between the study of joinings in ergodic theory and symmetric quantum channels in quantum information theory. Three additional papers beyond [BaCaMu14] addressing spin-off problems from this project, have been framed for completion in the next few years.

2.5. Connes’s Embedding Problem and Sofic Groups. In [Co76], Connes suggests that every type *II*₁ factor acting on a separable Hilbert space should admit a $*$ -embedding into an ultrapower of R . This “Connes Embedding Conjecture” has recently been shown by Marius Junge et. al. to be equivalent to a “matrix-valued” version of Tsirelson’s problem asking whether the quantum correlations prescribed by standard quantum mechanics agree with the correlations prescribed by quantum field theory [JuNaPaPeScWe11].

The above in the setting of group von Neumann algebras is closely related to a question of Gromov, asking whether every countable group G is sofic, roughly meaning one can approximate the group structure of G on finite patches using finite symmetric groups to arbitrary precision with respect to normalized Hamming distance [Pe08]. G. Elek and E. Szabo proved that if a group is sofic, then the Connes Embedding Conjecture holds for its group von Neumann algebra [ElSz06]. We note that the sofic property can be regarded as a weakening of Følner's condition for amenability, and so this problem is related to the general research program above.

It is now widely believed that Gromov's conjecture should be false. The more specialized question, perhaps attributed to Nate Brown, of whether every one-relator group is sofic (cf. open question 4.10 of [Pe08]). In [Ba11] we prove that a non-residually solvable one relator group constructed by Gilbert Baumslag is sofic, providing what is currently the "worst" known example of a sofic one-relator group.

2.6. Projective Spectrum in Banach Algebras. In the above discussion of the Fourier algebra, we noted that rigidity questions in group von Neumann algebras via the correspondence point of view admit a strong connection with the theory of Banach algebras. For the above reason, my research program is deeply sensitive to research aiming to find geometric invariants of Banach algebras. A few years ago I attended a seminar at SUNY Albany in which I was introduced to an elegant and subtle multivariable spectrum for Banach Algebras due to Rongwei Yang . In its most general form, the spectrum of a Banach algebra-valued holomorphic function f on a domain $\Omega \subset \mathbb{C}^n$ is the set of $z \in \Omega$ for which $f(z)$ is not invertible. This apparently small generalization is sensitive to multivariable complex analytic structure, since for any reasonable Banach algebra every path component of the associated resolvent is a domain of holomorphy. Even for linear f and commutative Banach algebras the spectrum is always hyperplane arrangement, and so is interesting geometrically. When I visited SUNY, Rongwei asked me if it is possible to compute the spectrum of the standard unitary generators of the free group factor. Using some deep ideas from free probability theory we were able to, within a few weeks, completely calculate that spectrum [BaCaYa11]. The general program of study here is now to determine precisely how the geometry is tied the structure of the Banach Algebra. A recent first step has been made in this direction by R. Yang and P. Cade, who established a connection between Connes's cyclic cohomology and the de Rham cohomology of the resolvent set [CaYa13]. We are currently pursuing several problems on the spectrum in order to better understand this result, from trying to make explicit connections with K-theory in the finite-dimensional setting to connecting the spectrum with complex foliations. This project promises to generate new results for many years.

2.7. Undergraduate Research Collaborations. At a small liberal arts college, research is valued primarily in terms of its ability to engage students outside the classroom. My personal approach has been to chip off subproblems from my major research work that (1) have the potential for impact, (2) can be framed in a concrete way for the student (3) admit immediate partial results with modest effort, and (4) can be placed in the context of the theory in the final writing of the student paper.

This has been reasonably successful, evinced by the fact that I am responsible for 6 of the 10 undergraduate research collaborations currently highlighted on Siena's mathematics department webpage.

2.7.1. *Explicit Formulas for the Multivariable Euler and Bernoulli Numbers.* In 2011, under my supervision Siena undergraduate Francesca Romano pursued a multivariable extension of work of David Vella to find new explicit formulas for Euler and Bernoulli numbers. Her work has been accepted for publication in the refereed journal *Involve* [Ro11]. She spoke about this work at the Union College Mathematics Conference and the National AMS/MAA Joint Meetings. Fran is now a mathematics PhD student at the University of Connecticut.

The Faa di Bruno formula for the coefficients of higher derivatives of a composite function can be understood in terms of set partitions. Set partitions are used for the computations of cumulants in probability theory. The translation of these calculations into the setting of free probability entails the passage from partitions to noncrossing partitions. One may naively hope to reverse engineer connections between free difference quotients or Fox's free differential calculus by examining an analogue of Faa di Bruno's formula in which the lattice of partitions is replaced by the sublattice of noncrossing partitions. Since freeness requires more than one variable, the first step in this program is to generalize the results of the above paper to higher dimensions, which Fran attempted in [Ro11].

2.7.2. *Moments in Finite von Neumann Algebras.* In 2010, Siena undergraduate Maureen Jeffery, Don Hadwin and I found new necessary conditions for the failure of the Connes Embedding Conjecture. This work was published in the refereed journal *Involve* [BaHaJe11]. Maureen is now a mathematics PhD student at Syracuse University.

The Connes Embedding Conjecture discussed above admits a number of undergrad-friendly approaches. One particular approach is to attempt to find a two-variable noncommutative polynomial $p(t_1, t_2)$ such that $Tr_k(p(A_1, A_2)) \geq 0$ for all positive definite contractions $A_1, A_2 \in M_k(\mathbb{C})$, for all $k \in \mathbb{N}$, and to show that for separable infinite-dimensional \mathcal{H} , whenever any positive invertible contractions x_1 and x_2 in $B(\mathcal{H})$ generate a finite von Neumann algebra with trace τ one has $\tau(p(x_1, x_2)) \geq 0$. If the first inequality holds and the second does not, then one has disproved the Connes Embedding Conjecture. For the undergraduate project, one proceeds by increasing degree and then matrix dimension has the student study the noncommutative polynomials and positivity condition on matrices. For the candidate polynomials we then introduce general operator algebra positivity arguments to establish the second inequality from the abstract point of view. Collaboration with Maureen Jeffery and Don Hadwin led to modest new necessary conditions in degree two for finding a counterexample of the Connes Embedding Conjecture.

2.7.3. *Non-residually solvable Hyperlinear One-Relator Groups.* In 2010, Siena undergraduate Nick Noblett and I extended results of [Ba11] to find new non-residually solvable hyperlinear one-relator groups. This work was published in *Involve* [BaNo10],

and was presented by Nick at the Hudson River Undergraduate Mathematics Conference.

The one-relator group studied in [Ba11] belongs to a class of one-relator groups that is still not known to contain only sofic groups. In the above undergraduate collaboration with Nick Noblett we were able to find identify other sofic one-relator groups in this class.

*2.7.4. The Burnside Group $B(3, 2)$ as a Two-Relator Quotient of $C_3 * C_3$.* In 2007, under my supervision Siena undergraduate Matthew Farrelly studied the structure of the (finite) free Burnside group $B(3, 2)$, employing the software GAP, resulting in a paper published in the Rose Hulman Undergraduate Mathematics Journal and a presentation at the Hudson River Undergraduate Mathematics Conference [Fa07].

Richard Kadison asked whether the group von Neumann algebras of infinite free Burnside groups are type II_1 factors. These groups arose as the first examples of nonamenable groups that do not contain nonabelian free subgroups. In my dissertation I noted, using the fact well-known to group theorists that the centralizer of any nonidentity element of one of these groups is finite, to answer this question in the affirmative. Recently it has been pointed out that this fact also obtains that the infinite Burnside groups do not have Murray and von Neumann's property Γ , which is a step in the right direction toward the famous open problem of whether these groups have property (T) .

The free Burside group $B(n, 2) = \langle a, b | g^n \rangle$ is known to be infinite if $n \geq 665$ and odd. It is still an open problem whether or not $B(5, 2)$ is finite. The group $B(3, 2)$ is a finite group, and I gave students the option of trying to prove this fact on a take-home modern algebra midterm. One of these students solved the problem, and continued the project over the summer, resulting in the above publication.

2.7.5. Orthogonal Pairs of MASAs in Matrix Algebras. In 2006, under my supervision Siena Undergraduate Andrew Warner and I studied orthogonal families of maximal abelian $*$ -subalgebras of matrix algebras. Andrew wrote a 27 page paper on this project that was never published. He recently earned a PhD in Mathematics from Rensselaer Polytechnic Institute.

One may regard $M_k(\mathbb{C})$ as a noncommutative probability space where matrices are analogous to random variables and the normalized trace τ is analogous to integration against the probability measure. In this setting two $*$ -subalgebras \mathcal{A} and \mathcal{B} of $M_k(\mathbb{C})$ are independent if $\tau(AB) = \tau(A)\tau(B)$ for all $A \in \mathcal{A}$ and $B \in \mathcal{B}$. The case where $(\mathcal{A}, \mathcal{B})$ is an independent pair of maximal abelian $*$ -subalgebras (masas) of $M_k(\mathbb{C})$ is of considerable interest even in the infinite-dimensional theory. Two such pairs are said to be isomorphic if a $*$ -automorphism of $M_k(\mathbb{C})$ sends one pair onto the other. Popa conjectured that for k prime that every pair of orthogonal masas in $M_k(\mathbb{C})$ is isomorphic to a pair of a very specific form, where the first entry is the algebra of diagonal matrices D_k and the second entry is uD_ku^* , where u is a special complex Hadamard matrix. For primes greater than 5 Popa's conjecture is false. Andrew and I worked through Uffe Haagerup's proof that Popa's conjecture holds for $k = 5$. This

work was never published, since it contained nothing really original (we first got our own proof, but you just can't beat Uffe).

2.7.6. *The Doubling Operator in Differential Equations.* My first undergraduate research project, working with Ryan Decker, explored existence and uniqueness of solutions of ordinary differential equations that incorporated the operator $Df = 2f$. His findings were presented at the Hudson River Undergraduate Mathematics Conference.

This was a more traditional “from the ground up” undergraduate research project, which still leaves almost an entire theory in need of development.

REFERENCES

- [Ba11] J. Bannon. *A non-residually solvable hyperlinear one-relator group*. Proc. Amer. Math. Soc. **139** (2011), 1409-1410.
- [BaCaMu14] J. Bannon, J. Cameron and K. Mukherjee. *Noncommutative joinings*, (in preparation)
- [BaCaWh14] J. Bannon, J. Cameron and S. White. *Weakly mixing correspondences and property (T) for II_1 factors*, (in preparation).
- [BaCaYa11] J. Bannon, P. Cade and R. Yang. *On the spectrum of Banach algebra-valued entire functions*. Illinois J. Math. **55** (2011), no. 4, 1455-1465.
- [BaFa11] J. Bannon and J. Fang. *Some remarks on Haagerup's approximation property*. J. Oper. Theory **65** (2011), no. 2, 403-417.
- [BaHaJe11] J. Bannon, D. Hadwin and M. Jeffery. *A note on moments in finite von Neumann algebras*. Involve **4** (2011), no. 1, 65-74.
- [BaNo10] J. Bannon and N. Noblett. *A note on non-residually solvable hyperlinear one-relator groups*. Involve **3** (2010), no. 3, 345-347.
- [BaRa07] J. Bannon and M. Ravichandran. *A Følner invariant for type II_1 factors*. Expo. Math. **25** (2007), no. 2, 117-130.
- [BeRo88] V. Bergelson and J. Rosenblatt. *Mixing actions of groups*. Ill. J. Math. **32** (1988) 65–80.
- [CaFaMu13] J. Cameron, J. Fang and K. Mukherjee. *Mixing subalgebras of finite von Neumann algebras*. New York J. Math. **19** (2013) 343-366.
- [CaYa13] P. Cade and R. Yang. *Projective spectrum and cyclic cohomology*. J. Funct. Anal, v 265 issue 9 (2013), 1916-1933.
- [ChCoJoJuVa01] P. Cherix, M. Cowling, P. Jolissaint, P. Julg and A. Valette. *Groups with the Haagerup property. Gromov's a -T-menability*. Progress in Mathematics **197** (2001), Basel: Birkhäuser Verlag.
- [Co76] A. Connes. *Classification of injective factors*. Ann. Math. **104** (1976), pp. 73-115.
- [Du08] R. Duvenhage. *Joinings of W^* -dynamical systems*. J. Math. Anal. Appl. **343** (2008), 175–181.
- [Du10] R. Duvenhage. *Ergodicity and mixing of W^* -dynamical systems in terms of joinings*. Illinois J. Math. **54** no. 2 (2010), 543-566.
- [Du12] R. Duvenhage. *Relatively independent joinings and subsystems of W^* -dynamical systems*. Studia Math. **209** (2012) 21-41.
- [Gl03] E. Glasner. *Ergodic theory via joinings*. Mathematical Surveys and Monographs, vol. 101, American Mathematical Society, Providence, RI, 2003.
- [ElSz06] G. Elek and E. Szabo. *On sofic groups*, J. Group Theory **9** (2006), 161–171.

- [Fa07] M. Farrelly. *The Burnside Group $B(3,2)$ as a Two-Relator Quotient of $C_3 * C_3$* , Rose Hulman Undergraduate Math. J. **8** (2007) no 2.
- [Ha44] P. R. Halmos. *In general a measure preserving transformation is mixing*. Ann. of Math. (2) **45** (1944), 786–792.
- [HLS81] R. Høegh-Krohn, M.B. Landstad and E. Størmer. *Compact ergodic groups of automorphisms*. Ann. Math. **114** (1981), 75–86.
- [JaPi01] V. Jakšić and C.-A. Pillet. *A note on eigenvalues of Liouvilleans* J. Statist. Phys. (2001) 937–941.
- [Ji13] Y. Jiang (<http://mathoverflow.net/users/9305/jiang>). *Find a lower bound for a pre-invariant $Fol(L(F_m), X_m)$* , <http://mathoverflow.net/questions/126644/find-a-lower-bound-for-a-pre-invariant-follf-m-x-m>.
- [Jo02] P. Jolissaint. *Haagerup approximation property for finite von Neumann algebras*. J. Oper. Theory **48** (2002), 549–571.
- [JuNaPaPeScWe11] M. Junge, M. Navascues, C. Palazuelos, D. Perez-Garcia, V.B. Scholz and R.F. Werner, *Connes’ embedding problem and Tsirelson’s problem*, J. Math. Phys. **52** (2011).
- [KePi08] D. Kerr and M. Pichot. *Asymptotic abelianness, weak mixing and property (T)*. J. Reine Angew. Math., **623** (2008), 213–235.
- [MvN36] F.J. Murray and J. von Neumann. *On rings of operators*. Ann. of Math. **37** (1936), 116–129.
- [MvN37] F.J. Murray and J. von Neumann. *On rings of operators II*. Trans. Amer. Math. Soc. **41** (1937), 208–248.
- [MvN40] F.J. Murray and J. von Neumann. *On rings of operators III*. Ann. of Math. **41** (1940), 94–161.
- [MvN43] F.J. Murray and J. von Neumann. *On rings of operators IV*. Ann. of Math. **44** (1943), 716–808.
- [Pe11] U. Pennig. *von Neumann algebras in physics: a rough sketch of algebraic quantum field theory*. http://wwwmath.uni-muenster.de/reine/u/upenn_01/slides/vonNeumannPhysics.pdf.
- [Pe08] V. Pestov. *Hyperlinear and sofic groups: a brief guide*. Bulletin Symb. Logic **14** issue 4 (2008) 449–480.
- [Po07] S. Popa. *Deformation and rigidity for group actions and von Neumann algebras*. International Congress of Mathematicians. Vol. I, Eur. Math. Soc., Zürich (2007), pp. 445–477.
- [Ro11] F. Romano. *More explicit fomulas for Euler and Bernoulli numbers*. to appear in Involve.
- [Ro48] V. Rokhlin. *A general measure-preserving transformation is not mixing*. Doklady Akad. Nauk SSSR (N.S.) **60** (1948), 349–351.
- [Ta38] A. Tarski. *Algebraische Fassung des Massproblems*. Fund. Math. **31** (1938), 47–66.
- [Wa72] M. E. Walter, *W^* -algebras and nonabelian harmonic analysis*, J. Funct. Anal. **11** (1972), 17–38.

DEPARTMENT OF MATHEMATICS, SIENA COLLEGE, 515 LOUDON ROAD, LOUDONVILLE, NY 12211

E-mail address: jbannon@siena.edu