TEACHING STATEMENT

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The value of a problem is not so much in coming up with the answer but in the ideas and attempted ideas it forces on the would-be solver.
-I.N. Herstein

1. Introduction

I believe that nothing we teach is more essential to our average student than the ability to solve a complex problem and communicate its solution in a precise and informal way. My main effort as an instructor is spent organizing unobtrusive experiences in which my students encounter and then engage "wellspring" problems that aim to build their mathematical maturity and world view. This approach goes by many names, such as IBL (Inquiry Based Learning), Discovery Learning or Modified Moore Method. My students spend most of their time working on problems and communicating their findings with others.

Over the years I have found that no matter how slowly and carefully we speak, the student is at a disadvantage, and that the best way to address this injustice is to give the student a head start. A student is not ready for an answer until he or she has asked a question. For this reason, I try to hold off on lecturing until *after* my students have done something on their own. My primary job in the classroom is to give well-placed hints and to critique student work. All of my students are required to learn to solve problems and communicate their ideas, and some of my students fall in love with the mathematics they find along the way and push things further. One of these outcomes without the other would be unacceptable to me, but seeing these outcomes together has made my teaching very rewarding.

Incoming freshmen are not exempt from this approach. In my Calculus classes I have successfully implemented a "flipped classroom": before each class meeting students are required to watch an online video or read a section of the textbook to prepare for class. At the beginning of class I pass out a short packet consisting of a one page "infographic" summarizing the main ideas and techniques of the video or reading, and a few good problems that encode the main ideas of the day. I divide the class into small groups and then move from group to group to field questions and give guidance. If a topic is particularly sticky the whole class works together to address the problem, always with a student leading the discussion at the board. In order to remediate student incoherence and algebra weakness, every solution must be written in complete English sentences that include a valid reason for each step.

Contemporary applications provide powerful ways to engage a student's prior experience, especially in the early stages of the student's career. For this reason, in

lower-level classes I often motivate abstract theory using an application of obvious public interest. Data science provides many such applications suitable for an introductory linear algebra course. Two examples of such applications are the Google Page Rank algorithm and Principal Component Analysis, the first of which uses eigenvalues and eigenvectors to rank webpages, and the second uses the spectral theorem for real symmetric matrices to analyze clustered data. A few well-designed projects or problems on cutting-edge applied topics like these give ample evidence that powerful mathematical ideas are behind the scenes in everyday life. Seeing the benefit of mathematical thinking and argument via applications, students gain appreciation and ability that scaffolds their study of more abstract topics.

I craft my upper-level mathematics courses as a microcosm for the community of research mathematicians. To illustrate how, I will describe below the format of my Introduction to Modern Algebra course and that of my Introduction to Real Analysis course.

In Modern Algebra, I use an extremely carefully crafted script of problems that leads students to discover basic results in the theory of groups. A student gains credit in the class each time he satisfactorily explains a problem to me. These conversations take place in class and during office hours, and although I do not refer to them as such, each conversation is a mini oral exam. During class, students work in small groups and when a student is ready to talk to me he includes his name on a list on the whiteboard. Throughout the class I visit each student on that list and give that student an opportunity to explain his solution. If he gives a correct solution and can answer all of my questions about the solution I allow him to show me his solution to another problem, repeating this until he makes a mistake or is stuck. I then give a hint and move on to the next student on the list. This policy sustains a vibrant economy in the class that keeps students working hard, but is much less confrontational than the original Moore Method of putting students on the spot in front of the class and sitting them down when they make an error. The frequent oral exams make it possible to prevent plagiarism, as those students who are mimicking their neighbors generally are doing so due to an inadequate understanding that is easily spotted and exposed with a targeted question. The structure of this course encourages students to learn to think about and talk mathematics informally with a colleague.

In Real Analysis, the emphasis shifts toward student writing and presentation skills. In addition to the "mini oral exam" format built around a careful problem script, students are required to carefully write up four to six problems per week to be submitted for grading. Three students per week are selected to each give a talk about one of their solutions in front of the class. The student talks are also graded. In contrast with the Algebra course, the structure of this course is meant to help teach students to write mathematics carefully and give a formal talk.

I usually offer the best students in my Algebra and Analysis classes an opportunity to do summer research with me. The outcomes from these research projects are discussed in my research statement, and are evidence that certain of my students have fallen in love with some mathematics and explored it more deeply. One of the six

student researchers has completed a PhD in Mathematics at Rensselaer Polytechnic Institute, another is a PhD student in Mathematics at Syracuse, and a third is a PhD student in Mathematics at the University of Connecticut. In an effort to make research opportunities even more widely available to our students, my colleague Brad Henry and I are writing a book *Knot Theory via Inquiry* that aims to bring students to current research in knot theory via a carefully crafted path of problems. Our work on this has been funded by the Academy of Inquiry Based Learning.

As I've written, it is not enough for me that a few of my students experience mathematics research. I want all of my students to learn to struggle with mathematics problems and learn to communicate what they discover. Here is a small sample of student comments about these upper-level classes that provide some evidence of engaged learning in my classroom:

- "Bannon is extremely approachable and loves the subject material. I liked the group style set up for learning as well to bounce ideas off of one another."
- "Very rarely were we just given a proof to do. I had to discover my own proofs for the assignments to move forward in the class."
- "I enjoyed the course and the material of Modern Algebra. I liked working individually at my own pace and fully understanding each concept to the best of my ability before moving on to the next concept. I also enjoyed receiving a little feedback at a time before being revealed to an entire solution to a problem."
- "I liked how much the course pushed us to think differently and do our best work. We only deserved a good grade when we truly understood what we were talking about."

Our lives are largely determined by how we respond to the problems we face, and for this reason the habits of mathematical research can improve the quality of our lives. My students practice these habits while gaining confidence in their problem solving ability and ownership of beautiful mathematical ideas. They engage in the struggle to strike the delicate balance of precision and informality in their writing and speaking. At least for a while, my students are mathematicians.

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